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Fractals in the Hypothalamus and Music

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Abstract:
A fractal is a mathematical shape that is infinitely complex, while its pattern repeats endlessly. Regardless of how zoomed in or zoomed out the analyses is achieved, the segment to study looks very similar in every part of the fractal to the whole picture. Nature is written in fractal language and in its logarithmic or exponential formulas. In this paper we will briefly study some linear and temporal fractals which might happen commonly for some hypothalamic functions and some parameters of music.

Keywords: Hypothalamus, music, fractal, brain, Suprachiasmatic Nucleus, SCN, heart, recursive, self-similarity, Mozart, Bach.

INTRODUCTION

The fractal concept was first introduced by the Mathematician Benoît Mahdelbrot in 1982 and it refers to a spatial or temporal characteristic phenomena which are continuous and having a fractal dimension repeated in a topological sense with a different orientation (Liu et al., 2003). The word fractal comes originally from Latin fractus, which means irregular, broken, made into pieces. In nature there are fractal structures everywhere which are often described as phenomenon of nature. But they also happen in the metabolic rhythms of living creatures. In mammals many of these fractals are stablished and controlled by our hypothalamus in the brain. However, the fundamental properties of these human physiological functions in fractals are needed to be described and defined in mathematical formalism, to a better understanding of its complexity for descriptive and predictive purposes (Bogdan et al., 2020). Some conceptual technicalities and new scientific vocabulary are being implemented for this aim, such as "dynamic functional connectivity" (DFC). This postulation has been created to get a more comprehensive approach to the reality of fractals in human metabolism, as a descriptive term for the multifractal analysis that might happen inside the human metabolism and its holistic functions (Bogdan et al., 2020). Mathematical defining fractals for some brain functions would undoubtedly benefit the Neuroscience field. Descriptive but also predictive values might be added to the Neuroscientific knowledge if, at some point, this theoretical and mathematical approach to the organic and changeable reality of the brain would be incorporated. Outside the biotic organisms, but processed and produced by the same, the music itself is having so often a fractal structure. Some compact musical pieces might be considered as mirrors of a much compounded but similar structure of small musical passages, which repeats in successive sequences with slightly different variations reminding like reflexes in a mirror the main musical version but under "different lights". The human working brain is able to distinguish between rhythm, tone and sound (Ventegodt et al., 2008) and in each of these musical dimensions, fractals are possible to happen. Recursive, iteration and self-similarity are common attributes in music to make its sound variations but also they are common resources to make fractals in geometrical and temporal spheres. Our interest in this paper is to prefatory prove how the brain or its metabolic viability and music might share fractal mathematics principles.

MATHS OF THE FRACTALS

There are two different fractals: "ideal" fractal (geometry in the space) and "natural" fractal (organic or temporal). The most imaginary, geometrical and exponential fractals is calculated with the basic formula of Mandelbrot: $\mathbf{Z = Z^m + C}$**,** where the exponential **m** is very variable changing the shape and **C** is a complex point in the plane that might be different depending on how many itineration have been required. Another standard method to measure ideal and spacial fractals is using the Hausdorff Dimension, which formula is **D = log N/ log s**, where **N** is the number of parts a fractal produces from each segment and **s** is the size of each new part compared to the original segment (fig. 1).

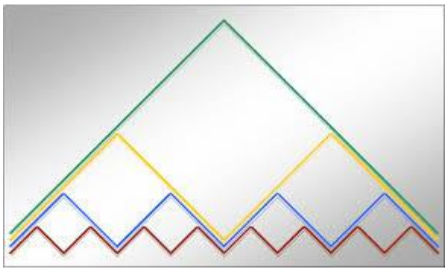

Figure 1: First lines of the Sierpinski triangle, a fractal set with the overall shape of an equilateran triangle, subdivided into smaller recursivelly equilateral triangles.

For natural, living and temporal fractals of time series the measurement would be made with the Hurst exponent (H) being the formula **D = 2 – H.** The Hurst exponent in a time series can be characterized as the following: $H < 0.5$ when time series is mean reverting. $H = 0.5$ when time series is a random geometric Brownian motion and persistence does not exist. Finally, the $H > 0.5$ happens when the time series is trending, not aleatory and persistence does occur. It is noticeable to mention the Hurst exponent was firstly calculated by the mathematician Harold Edwin Hurst for getting a reliable way of measuring the tides (afflux vs reflux) of the Nile River for human safety. Nature was the inspiration to capture and define the *aprioristic* random or stochastic event of overflows in Nile. To obtain the Hurst exponent, the total temporal data is subdivided into time intervals of equal duration and the average for each segment is calculated. A straight line is fitted to these average values and the slope of the line is calculated. The Hurst exponent is calculated from the slope of that straight line.

WHERE ARE FRACTALS

The structure of fractals is certainly hypnotic and mesmerizing since reminds the same architecture is present in different scales of analysis. That enables the human being to understand and outreach much larger and multiplex compositions. There are quite a number of examples of fractal morphologies in our own body, i.e., brain, eye, heart or kidney contain fractal structures. Some other examples of natural fractals are blood vessel branching, networks of the tracheobronquial tree, neural networks in the brain, the folds of intestine or choroidal plexus, the helix and spiral structure of the cochlea, etc. (Zueva, 2015) (fig. 2).

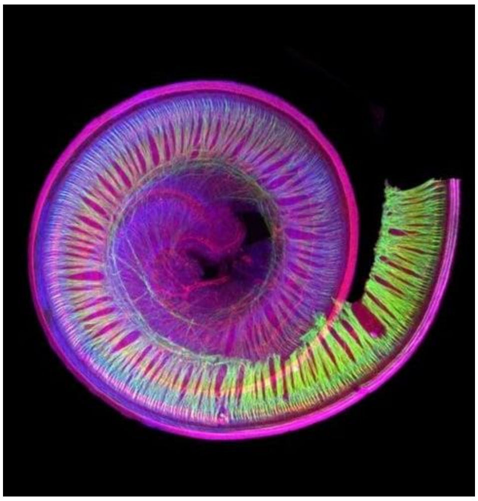

Figure 2: Spiral and fractal shape of the cochlea in the auditory system.

That is the case for the cerebellum and its skeleton. They possess highly fractal gyration that were proved and measured. The same fractality in cerebellum skeleton was found for men and women (Liu et al., 2003). This fractal appearance is very common in the brain (fig. 3).

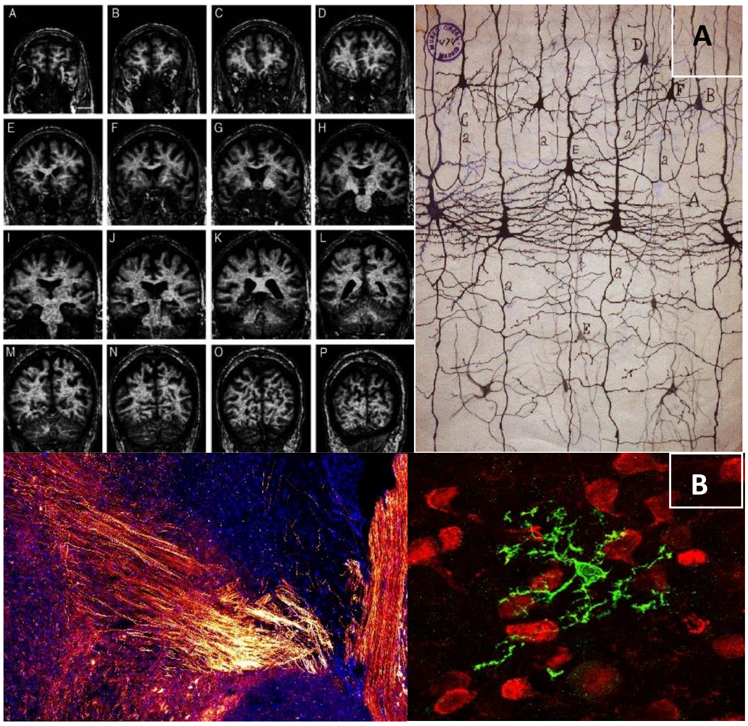

Figure 3: Pictures depict some biotic fractal pictures coming from the brain. The top pictures (A) show images of the human brain obtained with Magnetic Resonance Imaging (MRI) in a rostro-caudal approach. The other pictures belong to the Cajal´s drawings exploring the pyramidal neurons in the brain cortex. Both representations show an amazing symmetry and equilibrium between two hemispheres or adjacent neurons, reminding the fractal effect of images on a mirror. Pictures bellow (B) show real biological images of the brain, fibbers (pic on the left) and neurons (red) with astrocytes (green) (pic on the right) (images obtained with permission from the Icahn School of Medicine at Mount Sinai). It is noticeable how the fractal nature is more elusive or difficult to capture for naked eye in biological pictures (B) *vs* the pictures in A.

In the time-related dimension of the secular organism, the fractals also have a development. The constantly and regular rhythm of the heart generates robust fractal temporal organizations, in human and rodents. This similarity between species in temporal compositions is suggesting there might be a common scale invariant beat rate control mechanism between mammals (fig. 4).

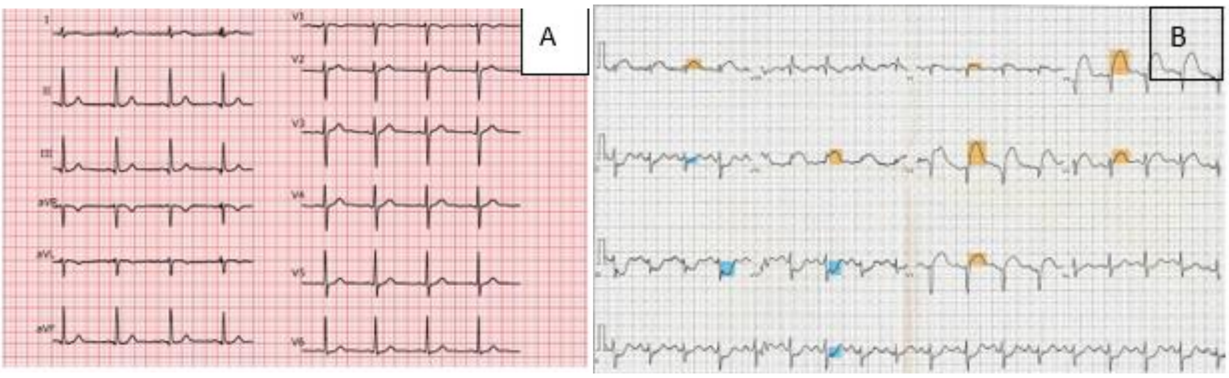

Figure 4: Measurement of the electrocardiogram (ECG) in human heart beat, for a normal state (A) and during a coronary infarction (B). It is noticeable how both cases are following a self-repeated pattern of rhythmic temporal fractal. From Wikipedia Commons public domain.

This regulated and ordered mechanism of response in the heart is mediated by the Suprachiasmatic Nucleus (SCN) in the brain. Lesions in the SCN of the brain in rats, without modifying the heart or its anatomy, can cause a larger scale exponent during heart beats (fig. 5). The same has been found in some human heart diseases with different causes (Hu et al., 2008). For the better adaptability, it might be important to identify other brain areas of control nodes (i. e. the complete dynamic functional connectivity, CDF) and their interactions to the SCN in the heart regulation. For instance, it has been proved other brain regions that are involved in fractal mammals neurophysiological heart regulation might be intergeniculate leaflet, midbrain raphe, paraventricular thalamus, limbic telencephalon or pedunculopontine/laterodorsally tegmental nuclei) (Hu et al., 2012; figs. 5 and 6).

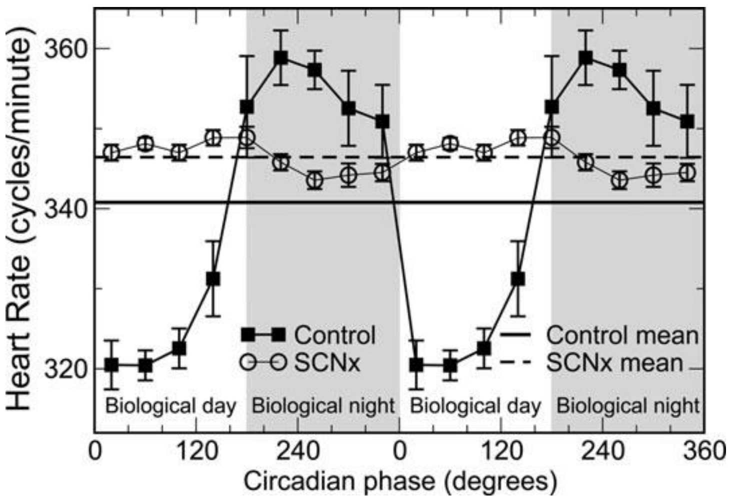

Figure 5: Graph showing the heart rate during day and night, for control and injured SCN rodents. It is noticeable how the damaged group does not have day/night variations and the level remins high as in night period (From Hu et al., 2008, with permission).

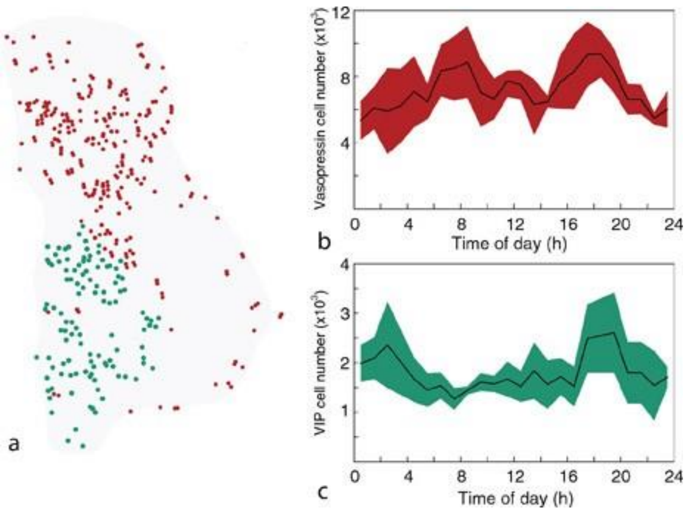

Figure 6: Pictures showing number of active cells expressing vasopressin (b) or vasoactive intestinal polipeptide (VIP) (c) during different periods of the day. It is noticeable how the activity reminds to the oscillantion rhythms of tides. There is a peak of activity for vasopressin is from 5-9h and from 15-20h, while the peak occurs for VIP during the time periods of 1-5h and 16-21h. (From Hofman and Swaab, 2002; with permission).

SYNAPSE AND ACTION POTENTIALS

Concerning to the shape of neurons and its activity [spontaneous synapses (fig. 7), long term potentiation or miniature synaptic transmission-mini)] some fractal dimensions have been defined. The long-term potentiation (LTP) was tested in the hippocampus (CA3-CA1 neurons) of recent born rodents and they found spontaneous and random releases of excitatory minis during consecutive traces which could be on a line for a fractal. The minis were firstly described 60 years ago and a sequence of them might provide information about multiple neuronal processes occurring at different time scales inside a presynaptic terminal, such as a fractal expression, like an echo (Lamanna et al., 2015).

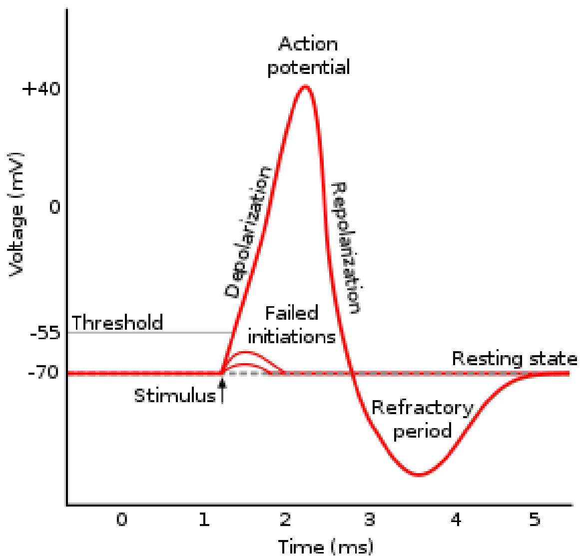

Figure 7: Visual representation of different stages of electrical activity through time (milliseconds) of a neuron synapse. The undulatory wave-shape and oscillating nature is very similar to those represented previously in figs. 4, 5 and 6, which are all capable of being subjected to calculation for obtaining the H exponent.

Referring to the anatomy of the neuron, the pyramidal neurons have been measured using fractal techniques to explore their complexity. Despite the fact their shapes are not strictly fractal (fig. 3A) the fractal definition was not in completely disagreement and it was increased its adjustability to a fractal concept with the longer length of the pyramidal dendrite (De Simoni et al., 2003).

FINE PLASTIC ARTS

Humans display a preference toward images with fractal-like statistic and appearance. The plastic superb artists Mauritius Cornellis Escher was able to increase the further mathematical and scientific research by anticipation or directed inspiration from his paintings. The fractal dimension in Escher work is plastically expressed with an admirable imagination and creativeness. His clever compositions were not easy understood at the time, because his oneiric and surrealistic but possible in real characterization of his new-fangled perspective were not well acquired or accepted those years. Recent investigations have proved the sense of aesthetic and/or beauty is not a product of the external stimuli (*i. e.* image properties, qualities of composition) but it is an individual act of perception through a complex sensory processing which implies very different levels of awareness in a dynamic functional connectivity for such stimuli (Swaab, 2013; Spehar et al., 2015; Swaab, 2019). That is the reason why beauty is in the eye of the beholder or "there is no accounting for taste".

LIFESPAN COGNITION

The perception of fractals is crucial to understand and generates hierarchical shapes in self-similarity. That will be required for language, grammar comprehension, general intelligence or sensitivity to visual complexity. Ontogenetically it commences to happen in the human development around 9 years of age (Martins et al., 2014). This ability to perceive and recognize fractals are related to the brain good functioning because they cause positive influences on cognitive skills (Zueva, 2015). During aging there are alteration in synaptic connectivity or plasticity and in Ca+ homeostasis which affects other metabolic processes. These modifications might have an effect over the quality of fractal perceptions. However, it has been proved a closer link to fine plastic arts and/or music created by great masters with grandiose brains might have a positive effect for cognitive abilities in fractal detection properties during lifespan. This could occur through a potential "curative impact", with healing and modifying human physiology thus its perception and/or interpretation of the world, through fine Arts (Zueva, 2015). In an *in vivo* study, conducted on and by humans, it was proved the brain fractal dimension, calculated from segmented cerebral white matter Magnetic Resonance Images (MRI), was correlating with greater fluid abilities for its brain activity. These capacities, in the subjects of the experiment, were considerable larger than expected by their childhood intelligence. That suggests brain possibilities and plasticity are happening beyond the childhood. In this research it was proved lifelong cognitive changes in fluid abilities were significantly associated with differences in neuroanatomical brain fractal white matter measurements. Curiously these results were happening with no relation to gender or the total white matter volume (Mustafa et al., 2012).

MUSIC

Music is a multidimensional environment, consisting of pitch, duration, the time interval between successive sounds, timbre, loudness, tempo, etc. It only happens in high evolved animals. Pythagoras identified the physics of intervals or distance between notes and that primary harmonic system is still used today (Wilson, 2003). It has been suggested that perception of certain external sounds, which are waves or pressure, may influence and modulate several biological functions such as blood pressure, heart rate, respiration, body temperature, cardiac and neurological functions. This positive or negative effect of music was tested with meditation music, mantra, kindness or hatred expressions, noises. Those waves might be inducing different responses to biological units in human metabolism depending on the classification of "good or bad" sounds (Lin et al., 2022). The ability to represent reality into hierarchical principles is thought to be possible because of a higher level of cognitive abstraction and recursive rules, which enables the language. It naturally easier to happens in musicians (84%) but it also occurs in non-musicians (without musical training) (71%) (Dias-Martin et al., 2017). This difference might suggest an important role of the auditory domain, besides of the visual domain, for building fractals representations in different dimensions (fig. 8).

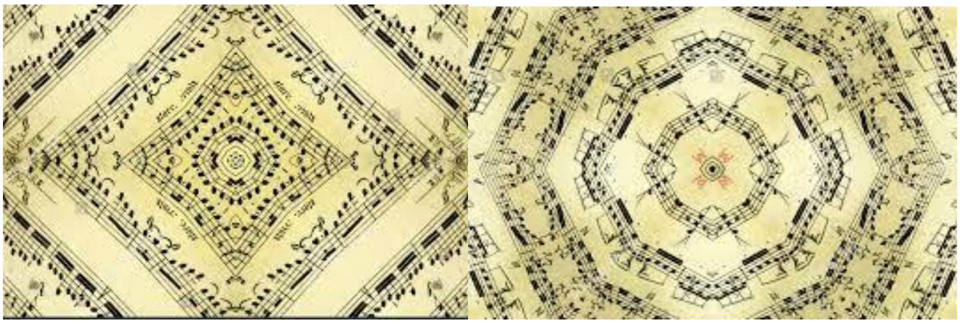

Figure 8: Metaphorical visual representation of the geometrical fractal dimension of music, in a kaleidoscope/mirror staging, where pieces of music sheet representing sounds are reflected and repeated.

It is believed that Wolfgang Amadeus Mozart wrote a piece of music in 1787 or 1788 as a joke, a duet for string, which could be read in two directions and both were perfectly beauty and coherent (harmonized) in terms of music ("*Der Spiegelkanon*") (fig. 9). Because of the facts that i) the key of G is reversible in the pentagram, ii) the notes are more suitable written inside the pentagram and iii) the five lines of pentagram are identical thoroughly, from top to down, the funny story and performance is possible. The music sheet is placed on a table between the two string musicians and each play what they see, from the beginning to opposite end of the sheet music. This aesthetic consideration of the fractal dimension of music, with this anagram, chiasmus or palindromic invertible canon, was only possible to be written with a deep knowledge of the nature of music and its inner meaning to our brain and auditory system. However, there are some doubts whether this piece was written by Mozart himself (Gardner, M. 1992).

Figure 9: Music sheet ascribed to Mozart named *Der Spiegel Duet* (the mirror). Because of the facts i) the key of G is reversible in the pentagram, ii) the notes are preferable written inside the pentagram and iii) the five lines of pentagam are identical from top to down, the joke is possible to be made and the song is successfully readable from both sides.

Many current and more pioneers than Mozart composers have assimilated this musical sense and their music sheets are really very close to the waves that happen in nature. Such are the cases of Johann Sebastian Bach (fig. 10).

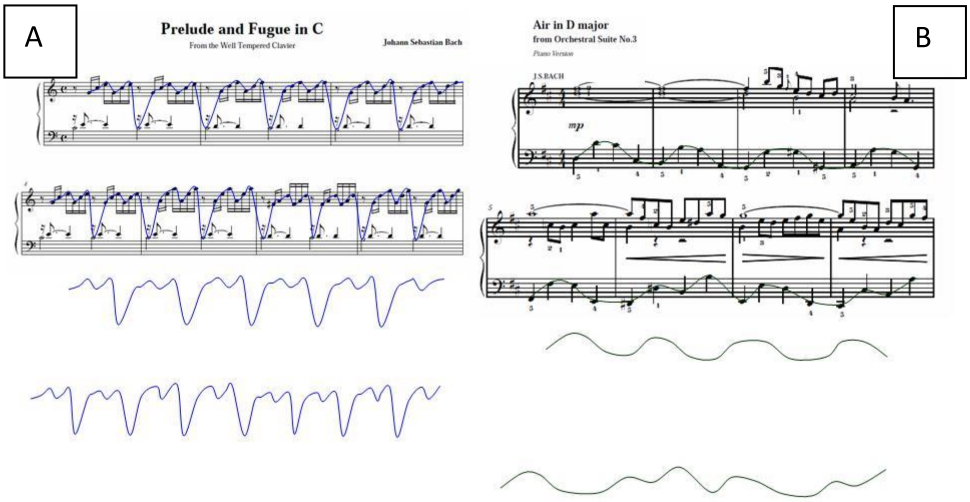

Figure 10: First pages of two well-known pieces composed by Bach. In the Prelude and Fugue in C (A) and in Air in D major (B) sequential and constant waves are apprehensible by the ear but also by the eye on the notes written (posterior lines). The oscillating lines in A reminds the heart beats of the human being, during a heart arrest (previously showed in fig. 4B and 5). Lines in B reminds the sound waves in the air which enters into our auditory system (fig. 11).

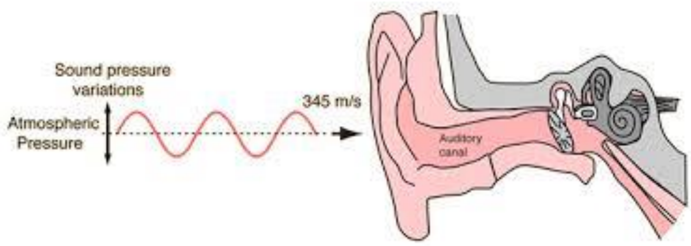

Figure 11: Graphical representation of the wave sound travelling through the air and entering into the human auditory system. (From Wikipedia Commons public domain).

SOUNDS UNDER WATER

It is well known dolphins and other cetaceans produce quite a number of sounds under the water to communicate, such as vocalizations, tonal whistles, clicks or burst pulses. This code plays a role in individual recognition ("signature whistles") and maintain group identity (Kershenbaum et al., 2013). In a study the hieratical temporal structure of 4 minutes of several pieces of sounds were compared: humpback whale song, human speech, rock music, symphonical classical music (fig. 12). As it is shown in fig., 12, solitary male humpback whales are able to sing long complex songs, like hermit thrushes, inside the musical meaning and definition of what a song is: introduction, chorus, common structural complexity in terms of self-similar "musical" groups (Kello et al., 2017).

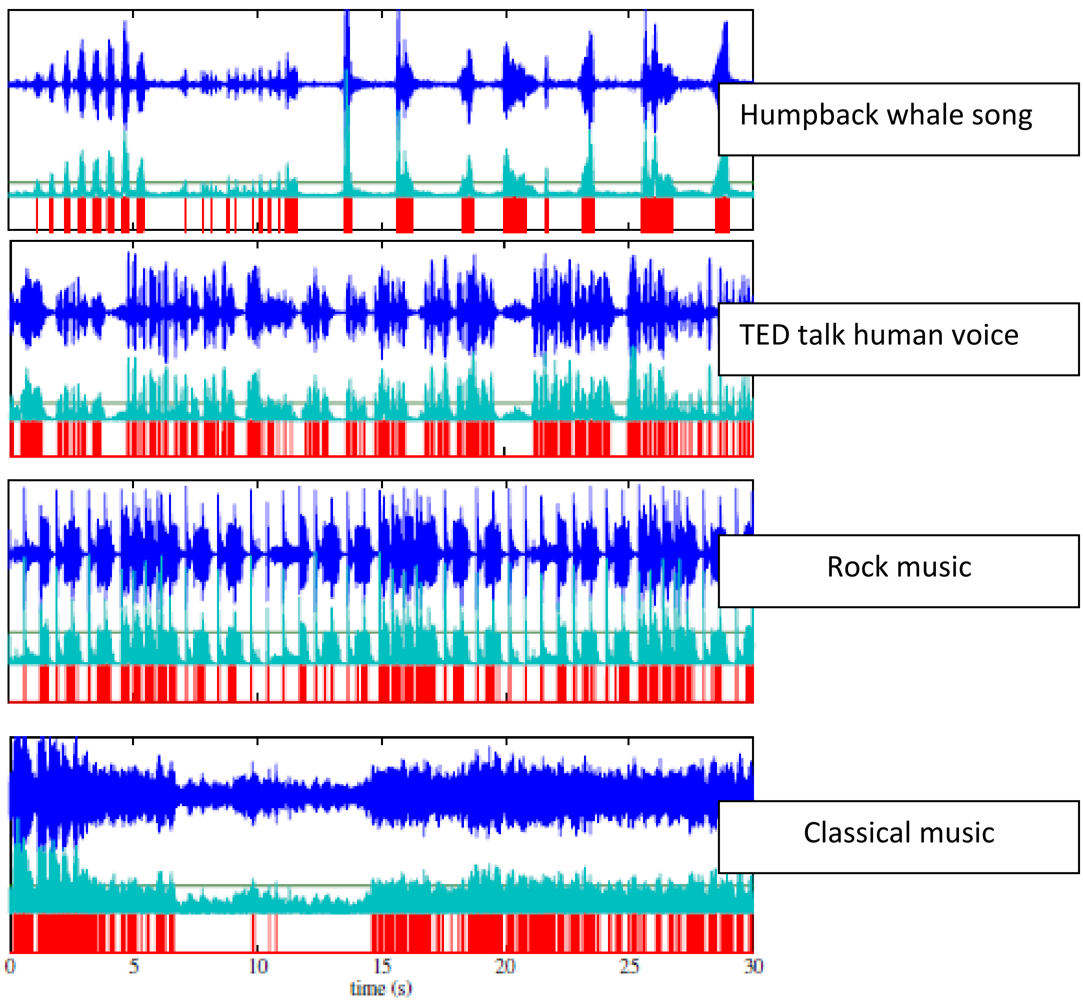

Figure 12: Different and specific statistical calculations of the weaves of sounds during 4 minutes of recording length. The blue in each of them and its shapes are depicting the

waves of sounds variations through time and space (air or water). The whale song shows a predictable pattern with a structure of a common song with sequences and repetitions (introduction, chorus, rhythm, etc) all under the water. This distribution is similar to those of the human which implies voices and communication (TED talk and rock music, "Back to Black" by ACDC), pointing out the presence of vocalizations in all those sounds. However, they are all different from classical music that is Brahms symphony number 4, where the musical structure is as fluid as sounds that are not vocalized (From Kello et al., 2017, with permission).

CONCLUSIONS

For an accurate calculation of Hurst exponent many data and records are required and in some cases this number is not achievable from biological samples or musical pieces with an end. However, the definition of fractals might worth it, when a reliable formula is obtained. That would enable the knowledge to predict human body reactions and getting a better understanding of great composers and plastic artist´s brain functions. In this brief text we have tentatively analysed two different pieces of music sheets and commented some body metabolic functions which show fractal auto-balanced and homeostatic expressions. In those cases when it would be possible, finding temporal parameters, calculating the average and mathematical slope definition of the straight line might add more accurately knowledge for some human metabolic responses or anatomical structures in Neuroscience. and musicians' grandiose brains might be better understood.

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REFERENCES

Bogdan, P., Eke, A., Ivanov, P. Ch. (2020): Editorial: Fractals and multifractals facets in the structure and dynamics of physiological systems and applications to homeostatic control, disease diagnosis and integrated cyber-physical platforms. Front. in Physiol. 11: 447.

De Simoni, A. Griesinger, CB., Edwards FA. (2003): Development of rat CA1 neurons in acute *versus* organotypic slices: role of experience in synaptic morphology and activity. J. Physiol. 550: 135-147.

Dias-Martin, M., Gingras, B., Puig-Waldmueller, E., Fitch, W. T. (2017): Cognitive representation of "musical fractals": Processing hierarchy and recursion in the auditory domain. Cognition. 161: 31-45.

Gardner, M. (1992): Fractal music, hypercards and more.... Mathematical Recreation from Scientific American Magazine. W. H. Freeman and Company. New York.

Hofman, M. and Swaab, D. (2002): A brain for all seasons. Cellular and molecular mechanism of photoperiodic plasticity. Progress in Brain Research, 88: 255-280.

Hu, K., Scheer, F. A. J. L., Buijs, Shea, S. A. (2008): The circadian pacemaker generates similar circadian rhythms in the fractal structure of heart rate in human and rats. Cardiovascular Research, 80: 62-68.

Hu, K., Meijer, J. H., Shea, S. A., VanderLeest, H. T., Pittman-Polletta, B., Houben, T., Oosterhout, F. V., Deboer, T., Scheer, F. A. J. L. (2012): Fractal patterns of neural activity exist within the Suprachiasmatic Nucleus and require extrinsic network interactions. PLoSONE, 7(11): e48927.

Kello, C. T., Bella, S.D., Médé, B., Balasubramaniam, R. (2017): Hierarchical temporal structure in music, speech and animal vocalizations: jazz is like a conversation, humpbacks sing like hermit thrushes. J. R. Soc. Interface. 14: 2017031.

Kershenbaum, A., Sayigh, L. S. And Janik, V.M. (2013): The encoding of individual identity in dolphin signature whistles: how much information is needed? PLoSONE, 8: e77671.

Lamanna, J., Signorini, MG., Cerutti, S.and Malgaroli A. (2015): A pre-docking source for the power-low behaviour of spontaneus quantal release: application to the analysis of LTP. Front. Cell. Neurosci. 9: 44.

Lin, C. D., Romano, P., Iliceto, S., Tona, F., Vitiello, G. (2022): On collective molecular dynamics in biological systems: a review of our experimental observations and theoretical modeling. Int. J. Mol. Sci. 23: 5145.

Liu, J. Z., Zhang, L. D. and Yue, G. H. (2003): Fractal dimension in human cerebellum measured by Magnetic Resonante Imaging. Biophysucal Journal, 85: 4041-4046.

Martins, M. D., Laaha, S., Freiberg, E. M., Choi, S., Fitch, W. T. (2014): How children perceive fractals: hierarchical self-similarity and coginitive development. Cognition. 133: 10-24.

Mustafa, N., Ahearn, T. S., Waiter, G. D., Murray, A. D., Whalley, L. J., Staff, R. T. (2012): Brain structural complexity and life course cognitive change. Neuroimage, 61: 694-701.

Spehar, B., Wong, S., van der Klundert, S., Lui, J., Clifford, C. W. G., Taylor, R. P. (2015): Beauty and the beholder: the role of visual sensitivity in visual preference. Frontiers in Human Neuroscience, 1-12.

Swaab, D. F. (2013): Art is Beauty in the eye of the beholder. Sikkens Foundation publishers. 1-32.

Swaab, D. F., (2019): Our creative brain. Atlas Contact. Uitgeverij.

Ventegodt, S., Hermansen, T. D., Kandel, I., Merrick, J. (2008): Human Development XIII: The Connection between the structure of the overtone system and the tone language of music. Some implications for our understanding of the human brain. The Scientific World Journal, 8: 643-657.

Wilson, R. J. (2003): Music and Mathematics. From Pythagoras to Fractals. Eds. John Fauvel, Raymond Flood and Robin Wilson. Oxford University Presss.