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Neutrosophic Structures in Statistical General Mathematical Functions

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Abstract:
The neutrosophic structures are very much relevant, vey essential and of course highly applicable to statistical and general mathematical concepts and structures. In practice. Sets in neutrosophic statistics are used, instead of crisp numbers in classical statistics. In addition, the neutrosophic concepts are undoubtedly very much applicable to a host of important mathematical ideals and concepts such as the transcendental functions and identities. In Classical Statistics all data are determined and this makes a very clear and vivid distinctions between neutrosophic statistics and classical statistics. In many cases, when indeterminacy is zero, neutrosophic statistics coincides with classical statistics. In many cases the neutrosophic can be used as a means for measuring the indeterminate data. Neutrosophic Data is the data that contains some forms of indeterminacy in this paper, efforts are intensified as much as possible to examine to some extent, the usefulness as well as the applicability of the concepts of neutrosophism in general mathematical functions and most especially in the area of problem solving.

Keywords: Neutrosophic Statistics, Classical Statistics, crisp numbers discrete neutrosophic data, continuous neutrosophic data, transcendental functions

INTRODUCTION

An extension of the classical statistics is the modern neutrosophic Statistics. In classical statistics the data is known and formed by crisp numbers while in neutrosophic statistics the data may have some forms of indeterminacies. Multiple problems such as the attributes in decision making processes are often being solved using the hesitant fuzzy linguistic information. In the case of the neutrosophic statistics, the data may be so direct. It may seem to be vague, ambiguous, incomplete, imprecise or even unknown. (Please, see [1]). In practice. Sets in neutrosophic statistics are used, instead of crisp numbers in classical statistics. In addition, the neutrosophic concepts are undoubtedly very much applicable to a host of important mathematical ideals and concepts such as the transcendental functions and identities.

ON NEUTROSOPHIC STATISTICS

In the neutrosophic statistics, the data may be ambiguous, vague, imprecise, incomplete, even unknown. Instead of crisp numbers used in classical statistics, one uses sets (that respectively approximate these crisp numbers) in neutrosophic statistics. (See [1])

Also, in neutrosophic statistics the sample size may not be exactly known (for example the sample size could be between 90 and 100; this may happen because, for example, the statistician is not sure about 10 sample individuals if they belong or not to the population of interest; or because the 10 sample individuals only partially belong to the population of interest, while partially they don't belong).

In this example, the neutrosophic sample size is taken as an interval n = [90, 100], instead of a crisp number n = 90 (or n = 100) as in classical statistics.

Neutrosophic Statistics refers to a set of data, such that the data or a part of it are indeterminate in some degree, and to methods used to analyze the data. (See [1])

In Classical Statistics all data are determined; this is the distinction between neutrosophic statistics and classical statistics.

In many cases, when indeterminacy is zero, neutrosophic statistics coincides with classical statistics.

We can use the neutrosophic measure for measuring the indeterminate data.

Neutrosophic Data is the data that contains some indeterminacy.

Similarly, to the classical statistics it can be classified as:

- discrete neutrosophic data, if the values are isolated points;for example: $6+i_1$, where $i_1 \in [0,1]$,7,$26+i_2$, where $i_2 \in [3,5]$;
- and continuous neutrosophic data, if the values form one or more intervals, for example: [0,0.8] or [0.1,1.0] (i.e., not sure which one).

Another classification:

- quantitative (numerical) neutrosophic data; for example: a number in the interval [2, 5] (we do not know exactly), 47, 52, 67 or 69 (we do not know exactly);
- and qualitative (categorical) neutrosophic data; for example: blue or red (we don't know exactly), white, black or green or yellow (not knowing exactly) (see [1]).

Also, we may have:

- univariate neutrosophic data, i.e., neutro-sophic data that consists of observations on a neutrosophic single attribute;
- and multivariable neutrosophic data, i.e., neutrosophic data that consists of observations on two or more attributes.

As a particular case we mention the bivariate neutrosophic data, and trivariateneutrosophic data.

A Neutrosopical Statistical Number *N* has the form: $N=d+i$,

where *d* is the determinate (sure) part of *N*, and *i*is the indeterminate (unsure) part of *N*.

For example, $a=5+i$, where $i \in [0,0.4]$, is equivalent to$a \in [5,5.4]$, so for sure $a \geq 5$ (meaning that the determinate part of a is 5), while the indeterminate part $i \in [0,0.4]$ means the possibility for number „a" to be a little bigger than 5.

While the Classical Statistics deals with determinate data and determinate inference methods only, the Neutrosophic Statistics deals with indeterminate data, i.e., data that has some degree of indeterminacy (unclear, vague, partially unknown, contradictory, incomplete, etc.), and indeterminate inference methods that containdegrees of indeterminacy as well (for example, instead of crisparguments and values for the probability distributions, charts, diagrams, algorithms, functions etc.

Neutrosophic Numbers of the form N = a+bI have been defined by W.B. Vasantha Kandasamy and F. Smarandache in 2003 [see 3], and theywere interpreted as "a" is the determinate part of the number N, and "bI"as the indeterminate In Imprecise Probability: the probability of an event is a subset T in [0,1], not a number p in [0, 1], what's left is supposed to be the opposite,subset F (also from the unit interval [0, 1]); there is no indeterminatesubset I in imprecise probability [see 4].

The function that models the Neutrosophic Probability of a random variable x is called *Neutrosophic distribution*: NP(x) = (T(x), I(x), F(x)), where T(x) represents the probability that value x occurs, F(x) represents the probability that value x does not occur, and I(x) represents the indeterminate / unknown probability of value x

I can deduce that the Neutrosophic idea is continuous within the interval while the crisp idea is discrete. And so, combining the cases we have as required and indicated: NP(x) = (T(x), I(x), F(x)) A true neutrosophic number contains the indeterminacy I with a non-zero coefficient.

Neutrosophic Real or Complex Polynomial.

A polynomial whose coefficients (at least one of them containing I) are neutrosophic numbers is called Neutrosophic Polynomials.

Similarly, we may have Neutrosophic Real Polynomials if its coefficients are neutrosophic real numbers, and Neutrosophic Complex Polynomials if its coefficients are neutrosophic complex numbers.

Proposition 1:
Let P(x) be a polynomial of degree 2. Then, thee exists two district solutions for P(x).

Proof: From [1] Let $P(x)$= (A + B.I) x^2 + (C + D.I) x + (E + F.I) = 0, and assume that the two solutions are given by $x_1 = a_1 + b_1I$ and $x_2 = a_2 + b_2I$ be the two neutrosophic real solutions of $P(x)$=0.

Then, set P(x) = (A+ B.I) [x – ($a_1 + b_1$I)]. [x- ($a_2 + b_2$I)] ≡(A + B.I) x^2 + (C + D.I) x + (E + F.I)

By equating the components, after the expansion of the. LHS, we have from the coefficients of x that C + D.I = - [A ($a_1 + a_2$) + (A ($b_1 + b_2$) + B ($a_1 + a_2$) + B ($b_1 + b_2$)) I], from where we would have that A ($a_1 + a_2$) = - C ……. (1) (A + B) ($b_1 + b_2$) + B ($a_1 + a_2$) = - D……… (2)

And from the constant term, we have, E + F.I = a_1a_2 + ($a_1b_2 + a_2b_1 + b_1b_2$) I, from where we have: a_1a_2 = E ………. (3), and $a_1b_2 + a_2b_1 + b_1b_2$ = F ……… (4).

We have $a_1 = E/a_2$, $a_1 + a_2 = - C/A$, $a_2 + E/a_2 + C/A = 0$.

Hence, $a_1 = \frac{-\frac{C}{A} \pm \sqrt{\frac{C^2}{A^2} - 4E}}{2}$ = $K_1 \in \mathbb{C}$ and $a_2 = E/a_1 = \frac{2E}{-\frac{C}{A} \pm \sqrt{\frac{C^2}{A^2} - 4E}}$ = $K_2 \in \mathbb{C}$.

Also, for b_1 and b_2 we have that $b_1 + b_2 = \frac{BC - AD}{A(A*B)}$ = $K_3 \in \mathbb{C}$. So, $b_2 = k_3 - b_1$.

Substituting this into (4), we have $k_1k_3 - k_1b_1 + b_1k_2 + k_3b_1 - b_1^2 = F$, so that $b_1^2 + b_1(k_1 - K_2 - k_3) - k_1k_3 - F = 0$ from where $b_1 = \frac{K_2 + K_3 - K_1 \pm \sqrt{K_1 - K_2 - K_3{}^2 + 4(K_1K_3 + F)}}{2}$ = $K_4 \in \mathbb{C}$, And $b_2 = K_3 - b_1 = K_3 - K_4 = K_5 \in \mathbb{C}$

By this, the following proposition is immediate:

Proposition 2:
A real polynomial equation of degree $n$possesses an n number of neutrosophic solutions in a neutrosophic.

Transcendental Functions
Transcendental function, in mathematics, a function not expressible as a finite combination of the algebraic operations of addition, subtraction, multiplication, division, raising to a power, and extracting a root. The transcendental functions are: Exponential functions, Trigonometric functions, Logarithmic functions, Inverse trigonometric functions. Examples include the functions log x, sin x, cos x, e^x and any functions containing them. (See [2])

For this aspect, we begin as follows:

The Exponential Function
Let $f(x) = e^x$. Here, we set x = a + bI

Addition

$$e^{(a_1+b_1I)} + e^{(a_2+b_2I)} = e^{(a_1+b_1I)} + e^{(a_2+b_2I)}$$

Multiplication

$$e^{(a_1+b_1I)}.e^{(a_2+b_2I)} = e^{(a_1+b_1I)+(a_2+b_2I)} = e^{(a_1+a_2)+(b_1+b_2)I)}$$

Division

$$\frac{e^{(a_1+b_1I)}}{e^{(a_2+b_2I)}} = e^{(a_1+b_1I)-\,(a_2+b_2I)} = e^{(a_1-\,a_2)+(b_1-b_2)I)}$$

Trigonometric Functions
Let A = $(a_1 + b_1I)$and B = $(a_2 + b_2I)$

Then, Sin (A $\pm$ B) = SinACosB$\pm$ CosASinB = Sin$(a_1 + b_1I)$Cos$(a_2 + b_2I) \pm$ Cos$(a_1 + b_1I)$Sin$(a_2 + b_2I)$ and Cos(A $\pm$B) = CosACosB$\mp$SinASinB = Cos$(a_1 + b_1I)$ Cos$(a_2 + b_2I)$ $\mp$Sin$(a_2 + b_2I)$Sin$(a_2 + b_2I)$

Moreover, let P = A + B = $(a_1 + b_1 I)$+$(a_2 + b_2 I)$ = $((a_1 + a_2) + (b_1 + b_2)I)$and Q = A – B = $((a_1 - a_2) + (b_1 - b_2)I)$

Then, sinP + sinQ = 2sin$\frac{1}{2}$(P+Q) cos$\frac{1}{2}$(P-Q), sinP-sinQ = 2cos$\frac{1}{2}$(P+Q) sin$\frac{1}{2}$(P-Q).

Also, cosP + cosQ = 2cos$\frac{1}{2}$(P+Q) cos$\frac{1}{2}$(P-Q), and cosP- cosQ = - 2sin$\frac{1}{2}$(P+Q) sin$\frac{1}{2}$(P-Q). For the tangent, tanA = sinA/cosB. So, let tan$\frac{1}{2}$A = t. Then, tanA = $\frac{2t}{1-t^2}$ = $\frac{2\tan\frac{1}{2}A}{1-\tan^2 A}$ = $\frac{2\tan\frac{1}{2}(a_1+b_1 I)}{1-\tan^2(a_1+b_1 I)}$ and tan(A± B) = $\frac{tanA \pm tanB}{1\mp tanAtanB}$ = $\frac{tan(a_1+b_1 I) \pm (a_2+b_2 I)}{1\mp tan(a_1+b_1 I)tan(a_2+b_2 I)}$

Logarithmic Functions
By the rule of logarithms, difine A and B as stated above, then, we would have that: logA + logB = logAB = log$(a_1 + b_1 I)(a_2 + b_2 I)$= log$(a_1 a_2 + (a_1 b_2 + a_2 b_1 + b_1 b_2)I)$ and logA - logB = logA/B = log$\frac{(a_1+b_1 I)}{(a_2+b_2 I)}$ = log($\frac{a_1}{a_2}$ + $\frac{a_2 b_1 - a_1 b_2}{a_2(a_2+b_2)}$ $.I$)(see [1])

Change of Base:
Let A = $(a_1 + b_1 I)$, B = $(a_2 + b_2 I)$and C = $(a_3 + b_3 I)$.

Then,

$$\log_B A = \frac{log_C A}{log_C B} = \frac{log_{(a_3+b_3 I)}.(a_1+b_1 I)}{log_{(a_3+b_3 I)}.(a_2+b_2 I)}.$$

APPLICATIONS

The neutrosophic ideas could be seen as having a great deal of importance and applications in analyzing the states, status and nature of any given mathematical structures and the way to make use of them in reality and general practical sense.

CONCLUSION

Neutrosophic and fuzzy science has been viewed from statistical as well as mathematical dimensions such as the transcendental functions and the like.

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CONFLICTS OF INTEREST

The author declares that there is no competing of interests.

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