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Fixed Point in Finite Neutrosophic Topological Metric Space

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Abstract:
Briefly, neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It relates to a general form of logic in which each proposition has separate values for truth, falsehood, and indeterminacy. This was formally discovered by Florentin Smarandache. For instance, the Neutrosophic sets (NS) have a significant role for clustering, denoising, segmentation, and classification in numerous medical image-processing's as well as their general applications. Motivated by the above, in this paper, we show that if Y (I) is a complete neutrosophic metric space in which the function f is a contracting mapping on the neutrosophic metric space Y (I), then, there exists one and only one point v in Y (I) such that f (v) = v

Keywords: Neutrosophic metric space, neutrosophic Euclidean space, contracting mapping, fixed point, Cauchy sequence, continuity

INTRODUCTION

In the recent years many articles on continuity in neutrosophic topological spaces have been studied. We implore our esteemed readers to consult our references (M. Parimala, et al, 2017, 2019 P. Mani etal 2018, A.A. Salama et al 2012, 2014, and B.C. Tripathy et al, 2013, 2014) for further details. In (B. C. Pal, et al, 2022), the authors introduced the notion of continuity via neutrosophic minimal structure space in a paper titled on continuity in minimal structure neutrosophic topological space. Clarification was also made on the notion of product minimal space in neutrosophic topological space. Investigations were thus made of some different basic properties of N_m-continuity in neutrosophic minimal structure space, such as composition of Nm-continuous functions, product of Nm-continuous functions in product neutrosophic minimal structure space. Our focus in this paper is to consider the fixed point in finite neutrosophic topological complete metric space.

The notions of neutrosophy, neutrosophic algebraic structures, neutrosophic duplet and neutrosophic triplet were introduced by Florentin Smarandache (Smarandache, 1998). On refined neutrosophic algebraic structures and in particular the neutrosophic groups, several developments were introduced by Agboola Adesina (Agboola, 2015). After the successful feat, many researchers have as well tried to establish and studied further more on the refined neutrosophic algebraic structures. (Agboola etal, 2020). Further studies on refined neutrosophic rings and refined neutrosophic subrings, their presentations and fundamental were also worked upon. Also, Agboola, in his paper (Agboola, 2020) has studied the refined neutrosophic quotient groups, where more properties of refined neutrosophic groups were presented and it was shown that the classical isomorphism theorems of groups do not hold in the refined neutrosophic groups. The existence of classical morphisms between refined neutrosophic groups G (I1; I2) and neutrosophic groups G(I) were established. The readers can as well consult (Agboola et al, 2011, 2012; Bera et al, 2017; Smarandache et al, 2006, 2013, 2022 and seymour lipschutz, 1965) in order to have detailed knowledge concerning the refined neutrosophic logic, neutrosophic groups, refined neutrosophic groups and neutrosophy, in general.

PRELIMINARIES

Definition 2.1. (Agboola, 2020)

Suppose that (X (I_1; I_2); +;.) is any refined neutrosophic algebraic structure. Here, + and. are ordinary addition and multiplication respectively. Then I_1 and I_2 are the split components of the indeterminacy factor I that is $I = \alpha_1 I_1 + \alpha_2 I_2$ with $\alpha_i \in C$; $i = 1; 2$.

Definition 2.2. (Agboola, 2020)

Suppose that (G; $*$) is any group. Then, the couple (G (I_1; I_2); $*$) can be referred to as the refined neutrosophic group. Furthermore, this group can be said to be generated by G, I_1 and I_2. Hence, (G (I_1; I_2); $*$) is said to be commutative if $\forall x; y \in G$ (I_1; I_2), depicts that $x * y = y * x$. Otherwise, (G (I_1; I_2); $*$) can be referred to as a non-commutative refined neutrosophic group.

Theorem 3.1 (Agboola, 2020)

(1) Every refined neutrosophic group is a semigroup but not agroup. (2) Every refined neutrosophic group contains a group.

Corollary 2.1. (Agboola, 2020)

Every refined neutrosophic group (G (I_1; I_2); +) is a group.

Definition 2.3 (Agboola, 2020)

Let (G (I_1; I_2); $*$) be a refined neutrosophic group and let A (I_1; I_2) be a nonempty subset of G (I_1; I_2). A (I_1; I_2) is called a refined neutrosophic subgroup of G (I_1; I_2) if (A (I_1; I_2); $*$) is a refined neutrosophic group. It is essential that A (I_1; I_2) contains a proper subset which is a group. Otherwise, A (I_1; I_2) will be called a pseudo refined neutrosophic subgroup of G (I_1; I_2).

Definition 2.4 (Agboola, 2020)

Let H (I_1; I_2) be a refined neutrosophic subgroup of a refined neutrosophic group (G (I_1; I_2) ;). Define x = (a; bI_1; cI_2) ∈ G (I_1; I_2).

Theorem 2.2 (Agboola, 2020)

Let (G (I_1; I_2); +) be a refined neutrosophic group and let ($G(I)$; +) be a neutrosophic group such that where $I = xI_1 + yI_2$ with $x; y \in$ C. Let φ: G (I_1; I_2) $\rightarrow G(I)$ be a mapping defined by ϕ ((a; xI_1; yI_2)) = (a; ($x + y$) I) $\forall$ (a; xI_1; yI_2) ∈ (G (I_1; I_2) with $a; x; y \in G$: Then ϕ is a group homomorphism.

FIXED POINT IN FINITE NEUTROSOPHIC TOPOLOGICAL COMPLETE METRIC SPACE

Definition 3.1

Suppose that Y(I) is a neutrosophic metric space which involves a continuous mapping on Y(I) such that for every point in Y(I), there is a maintenance of contraction, then, the mapping f is continuous at each point in Y(I).

Definition 3.2

Suppose that Y(I) is a finite neutrosophic metric space. A neutrosophic function f: $Y(I) \rightarrow Y(I)$ is called a contracting mapping provided there is a real number γ for which $0 \leq \gamma < 1$ such that for every $x, y \in Y(I), d(f(x), f(y)) \leq \gamma d(x,y) \leq d(x,y)$. The implication here, is that the distance between the images of any two points is actually less than the distance between the points.

Proposition 3.1

Let Y (I) be a metric space and f: $Y\ (I)\ \rightarrow\ Y\ (I)$, a contracting mapping on Y (I) and let f be a contracting map on a complete metric space Y (I), say $d\ (f\ (x), f\ (y))\ \leq\ \beta d\ (x, y), 0\ \leq\ \beta\ <\ 1$ There exists one and only one point p in Y (I) such that $f\ (v)\ =\ v$.

Proof

Let a_o be any point in Y (I). Set $a_1 = f\ (a_0), a_2 = f\ (a_1) = f\ _2(a_0),\cdots, a_n = f\ (a_n - 1) = f_n(a_0),\cdots$ Claim: $\langle a_1,\ a_2,\cdots\rangle$ is a Cauchy sequence. It should be properly noted here that $d(f^{s+t}(a_o), f^t(a_o))\ \leq\ \beta d(f^{s+t-1}\ (a_o), f^t - 1(a_0)) \leq\cdots\leq \beta^t d(f^s(a_0), a_0)\ \leq\ \beta^t[d\ (a_0, f\ (a_0))\ +\ d(f\ (a_0),\ f\ ^2(a_0))\ +\cdots + d(f^{s-1}(a_0), f^s(a_0))$ But then, $d(f^{i+1}(a_0), f^i(a_0))\ \leq\ \beta^i d(f\ (a_0), a_0)$,

so, we have, $d(f^{s+t}(a_0), f^t(a_0))\ \leq\ \beta^t d(f\ (a_0), a_0)(1\ +\ \beta + \beta^2 +\ + \beta^{s-1})\ \leq\ \beta^t d\ (f\ (a_0), a_0)$ [1/6].

This is because $(1\ +\ \beta\ +\ \beta^2\ +\cdots+ \beta^{s-1})\ \leq\ 1/6$

Now, if we let e > 0 and set

$$\alpha = \begin{cases} e(1-\ \beta) if\ d(f(a_o),\ a_o) = 0 \\ \text{and} \\ \frac{e(1-\ \beta)}{d(f(a_o), a_o)}\ if\ d(f(a_o),\ a_o) \neq 0 \end{cases}$$

Now, since β > 0, there exists $n_o\ \in N$ such $a_{no} < \beta$. Hence, if $r \geq s > n_0$, then $d\ (a_s, a_r)\ \leq\ \beta_s$ [1/6] $d\ (f(a_0), a_0) < d$ [1/6] $d\ (f(a_0), a_0) \leq e$

And so $\langle a_n\rangle$ is a Cauchy sequence. Now, $Y\ (I)$ is complete and so $\langle a_n\rangle$ converges to a point p in Y (I) say. We then show here, that $f\ (v) = v$, for f is continuous and hence sequentially continuous, so,

$$f\ (v)\ =\ f\ (\lim_{\infty\rightarrow n} a_n)\ =\ \lim_{\infty\rightarrow n} f\ (a_n) = \lim_{\infty\rightarrow n} a_{n+1} = v$$

Finally, we show here that p is unique

So, suppose that $f(v) = v$ and $f(q) = q$ then $d\ (p, q) = d\ (f(v), f^i(q)) \leq \beta d\ (p, q)$ But then $\beta <$ 1, hence $d\ (p, q) = 0$. That is $p = q$.

Let $Y\ (I)$ be a metric space and let $f\!: Y\ (I) \longrightarrow Y\ (I)$ be a continuous mapping on $Y\ (I)$ i.e., there exists $y \in R$, $0 \leq y < 1$, such that for every $a, b \in Y$, $d\ (f(a), f(b)) \leq yd\ (p, q)$. f is continuous at each point $y_o \in Y\ (I)$.

For let $e > 0$ be given, then $d\ (y, y_o) < e, \Rightarrow d\ (f(y), f(y_o)) \leq yd(y, y_o) \leq ye < e$. Therefore, f is continuous.

Instances

If f is a function on the neutrosophic Euclidean $2 - space$, R^2 that is if $f: R^2 \rightarrow R^2$ is defined by $f(u) = ½ u$, then, f is contracting.

Applications

If Y(I) happens to be a complete neutrosophic metric space then this particular result is well applicable in various mathematical analysis.

CONCLUSION

We have been able to successfully show the contracting property as well as the uniqueness of the given fixed point in every complete neutrosophic metric space

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CONFLICTS OF INTEREST

The author declares that there is no competing of interests

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