



# Information Dynamics in All Japan Women Championship in High School Soccer 2020 Final

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## Abstract:

This paper is concerned with information dynamics in all Japan women championship in high school soccer 2020. The final game has been successfully simulated and modelled in terms of advantage and certainty of game outcome depending on the time. In the data analyses, as evaluation function scores, in addition to goal, it is realized that consideration of shoot, corner kick, or penalty kick are essential, and thus faithful analyses to games have been conducted, where proper choice and assessment of evaluation function scores are carefully made during the games. The refined game information depending on the time, makes possible for supervisor, coach, player or fan to reflect the past game and to prepare for the future game, so that the present analyses add a pedagogical value to activities relating to soccer other than entertainment.

*Keywords: Pedagogical Game, Soccer, Evaluation Function Score, Advantage, Certainty of Game Outcome*

## INTRODUCTION

Serious game is a game designed for a primary purpose other than pure entertainment, enjoyment or fun [1]. Serious games are a sequence of storytelling, where the idea shares aspects with game simulation, but explicitly emphasizes the added pedagogical value of fun and competition. The goal of a serious game is to support learning/training for educator or nurse. Supervisor of a soccer team is educator for the members: Daily activities in soccer support players, for they can learn importance of fairness, cooperation, harmony or patience under the rules. It is known that ultimate goal of soccer is to bring up lady and gentleman, but not to win or to enjoy the game. Thus, it is evident that soccer is a typical serious game.

Making use of game design patterns, Kelle et.al. [2] have implemented information channels to simulate ubiquitous learning support in an authentic situation. Lindley & Sennersten [3]'s schema theory provides a foundation for the analysis of game play patterns created by players during their interaction with a game. Lindley & Sennersten [4] have proposed a framework which is developed not only to explain the structures of game play, but also to provide schema models that may inform design processes and provide detailed criteria for the design patterns of game features for entertainment, pedagogical and therapeutic purposes.

Fullerton et al. [5] argue in favor of 'iterative' design method, which relies on inviting feedback from players early on, where the word 'iterative' refers to a process in which the game is designed, tested, evaluated and redesigned throughout the project. As part of this approach, designers are encouraged to construct first playable version of the game immediately after brainstorming and this way get immediate feedback on their ideas [5]. Play-testing, which lies in the heart of iterative approach, is probably most established method to involve players in design. Play-testing is not

primarily about identifying the target audience or tweaking the interface, but it is performed to make sure that the game is balanced, fun to play, and functioning as intended [5].

Game Ontology Project [6] is distinct from design rules and design patterns approaches that offer imperative advice to designers. It intends not to describe rules for creating good games but rather to identify the abstract commonalities and differences in design elements across a wide range of concrete examples. Rather than develop definitions to distinguish between games and non-games or among their different types, it focuses on analyzing design elements that cut across a wide range of games. Its goal is not to classify games according to their characteristics and/or mechanics [7] but to describe the design space of games.

Knowledge about game designs and game play patterns has grown fairly well, but little advancement has been made to clarify game progress patterns, which denotes how information of game outcome varies depending on time [8,9]. Currently the information dynamic model [8, 9], only makes it possible to treat and identify game progress patterns depending on time. The usefulness of the information dynamic model has been well documented, and successfully applied to American football [14], baseball [9], effect of medicine [10], soccer [11], shogi [13], or judo [8]. On the contrary, von Neumann's game theory [15], for example provide only a few outcomes due to each the decision by players.

The information dynamic model [8, 9] has been applied to soccer [11], in which only goal(s) is considered as the evaluation function score. However, it becomes clear that number of goal(s) is normally too few to conduct adequate analyses for soccer games, so that it is necessary to seek for the other candidate(s) of the evaluation function score to refine the analyses.

Main purpose of the present study is to simulate and model a soccer game, 9th all Japan High School Women Soccer Championship in Noevia Stadium Kobe on 10th January 2021 to support supervisor, coach, player or fan, in terms of advantage, certainty of game outcome together with the information dynamic model.

## METHOD OF ANALYSES

For clarity, elemental procedures for obtaining the advantage  $\alpha$ , certainty of game outcome  $\xi$ , and uncertainty of game outcome  $\varsigma$  will be explained by using a soccer game between teams A and B.

The advantage  $\alpha$  is defined as follows: When the total goal(s) of the two teams at the end of game  $GT \neq 0$ ,

$$\alpha = [GA(\eta) - GB(\eta)]/GT \text{ for } 0 \leq \eta \leq 1, \quad (1)$$

where  $GA(\eta)$  is the current goal sum for team A(winner),  $GB(\eta)$  is the current goal sum for team B(loser), and  $\eta$  is the normalized time, which is normalized by the total time.

When  $\alpha > 0$ , team A (winner) gets the advantage against team B (loser) in the game, while when  $\alpha < 0$ , team B (loser) gets the advantage against team A(winner). It is certain that when  $\alpha = 0$  the game is balanced. Note that goal is only one of the evaluation function scores in soccer, but in addition to goal, shoot, penalty kick, free kick and corner kick are candidates of the evaluation function score. It is critical how assessor(s) chooses and assesses the evaluation function score

during the game, as to be discussed. When the total goal(s) of the two teams at the end of game  $GT = 0$ ,  $\alpha=0$  for  $0 \leq \eta \leq 1$

The certainty of game outcome means what extent the game outcome (i.e., win or loss) is certain depending on time during the game, and the extent is given by the normalized value ranging from 0 to 1. The word 'certainty' is replaceable with 'probability' without loss of generality, but the accustomed word may be preferable. The certainty of game outcome  $\xi$  during the game is defined as follows: When the total goal(s) of the two teams at the end of game  $GT \neq 0$ ,

$$\xi = |GA(\eta) - GB(\eta)|/GT \text{ for } 0 \leq \eta < 1 \text{ (normal game) or } 0 \text{ (draw game) for } \eta = 1. \quad (2)$$

At  $\eta=1$ ,  $\xi$  is assigned to the value of 1, for at the end of game the information on the game outcome must be 100%. The reason why we take the absolute value of the advantage  $\alpha$  to get the certainty of game outcome  $\xi$  for  $0 \leq \eta < 1$  is that  $\xi$  is independent of the sign of  $\alpha$ . As far as the absolute value of the advantage  $\alpha$  increases (decreases), the certainty of game outcome  $\xi$  must increase (decrease). In case of draw game,  $\xi$  may be assigned to the value of 0 at the end of game  $\eta=1$ . When the total goal(s) of the two teams at the end of game  $GT=0$ ,  $\xi=0$  for  $0 \leq \eta \leq 1$ .

The uncertainty of game outcome  $\zeta$  during the game is defined as follows

$$\zeta = 1 - \xi. \quad (3)$$

This equation denotes that the current uncertainty of game outcome  $\zeta$  can be obtained by subtracting the current certainty of game outcome  $\xi$  from value of 1. Keeping in mind the forgoing elemental procedures to obtain the advantage  $\alpha$ , and certainty of game outcome  $\xi$ , it may be straight forward to apply them to actual soccer games. In soccer, goal must be as one of evaluation function scores, for it is critical factor for the game. However, there must be the other candidates such as shoot, corner kick, penalty kick, or free kick. In computer chess, evaluation function score for each of the moves has been successfully assessed by a computer based on objective human evaluation function scores that are adopted for the data analyses of human and computer interaction in shogi between Kunio Yonenaga (Shogi Meijin) and Bonkras (Computer Shogi World Champion in 2012) [13].

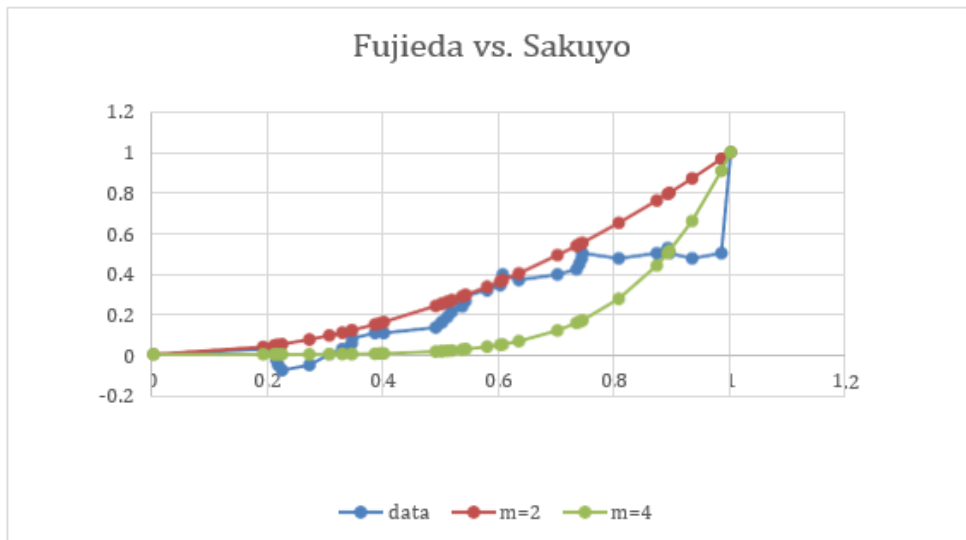
### CASE STUDY

In this section, investigated are a soccer game, which are the Final, Fujieda vs. Sakuyo of the 9th all Japan High School Women Soccer Championship in Noevir Stadium Kobe on 10th January 2021. This part is concentrated with the case study of Fujieda vs. Sakuyo. Table 1 summarizes results of data analyses for the game. The total time is 90 minutes (54,000 seconds), for the Final is 45 minutes half, and the total evaluation function scores are 38, where each of the shoot (including goal case), corner kick, free kick is counted one as evaluation function score in the present analyses. For example, shoot may seem to be very different from corner kick, but if one looks shoot and corner kick from the assessor's view point, they contribute to the game outcome to the same degree sometimes. At this point, it may be worth noting that occasionally a ball kicked from the corner goes into the goal directly. This is because corner kick is treated exactly same evaluation function score as shoot. When only goal is considered as the evaluation function score, few numbers may cause difficulty to analyze the game properly. Table 1 summarizes game records for the Final. In the game, after the kick off by Sakuyo, Fujieda gets the advantage until  $\eta=0.190$ . During this period, Fujieda makes one shoot, but since Sakuyo gets one corner kick at

$\eta=0.209$ , the game is balanced. However, from  $\eta=0.213$  to  $0.270$ , Sakuyo attacks to Fujieda, so Sakuyo increases the advantage  $\alpha$  with increasing the normalized time  $\eta$ . Then, Fujieda gets corner kick and shoot at  $\eta=0.305$ , and as the result the game is balanced again. From this time, Fujita keeps the advantage against Sakuyo, in such a way that the former increases the advantage  $\alpha$  with increasing  $\eta$  until the end of game. This denotes that Fujieda keeps  $\alpha > 0.05$ , when  $\eta > 0.343$ : For over 65.7% of the total time,  $\alpha$  is greater than  $0.05$ . Thus, this game can be defined as non-categorized game, though it might look as one-sided game.

**Table 1 Game records for the Final:Fujieda vs. Sakuyo**

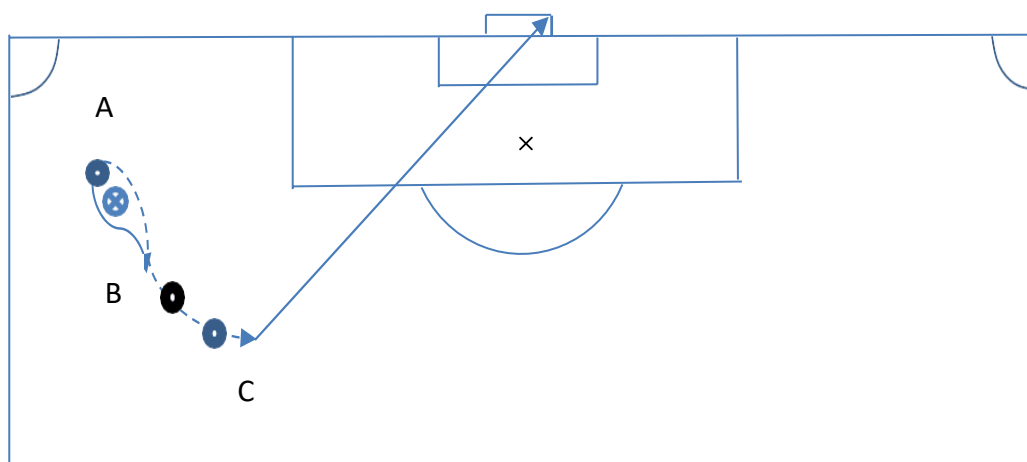
$\eta$	G <sub>A</sub>	G <sub>B</sub>	G <sub>A</sub> -G <sub>B</sub>	$\alpha$	Fujieda	Sakuyo
0	0	0	0	0		
0.19	1	0	1	0.0263	S	
0.209	1	1	0	0		C
0.213	1	2	-1	-0.0263		S
0.216	1	3	-2	-0.0526		S
0.223	1	4	-3	-0.0789	C	
0.27	2	4	-2	-0.0526	S	
0.305	4	4	0	0	C, S	
0.327	5	4	1	0.0263	S	
0.343	6	4	2	0.0526	C	
0.344	7	4	3	0.0789	S	
0.384	8	4	4	0.105	F	
0.393	9	4	5	0.132	S	
0.399	9	5	4	0.105	S	
0.489	10	5	5	0.132	S	
0.499	11	5	6	0.158	C	
0.5	11	5	6	0.158		
0.509	12	5	7	0.184	C	
0.517	13	5	8	0.211	G	
0.535	14	5	9	0.237	C	
0.54	15	5	10	0.263	S	
0.541	16	5	11	0.289	S	
0.578	17	5	12	0.316	G	
0.601	18	5	13	0.342	F	
0.605	20	5	15	0.395	S, S	
0.633	20	6	14	0.368		F
0.7	21	6	15	0.395	G	
0.733	22	6	16	0.421	C	
0.738	23	6	17	0.447	C	
0.742	24	6	18	0.474	S	
0.743	25	6	19	0.5	C	
0.806	25	7	18	0.474		S, S
0.872	26	7	19	0.5	S	
0.891	27	7	20	0.526	S	
0.894	27	8	19	0.5		S
0.933	27	9	18	0.474		S
0.984	28	9	19	0.5	F	
1	28	9	19	1		



**Figure 1: Certainty of game outcome  $\xi$  against normalized time  $\eta$**

Figure 1 shows the relation between certainty of game outcome  $\xi$  and normalized time  $\eta$ . The certainty of game outcome  $\xi$  increases with increasing the normalized time  $\eta$  except for the two points at  $\eta=0.209$  and  $\eta=0.305$  in the middle of game.

During this game, several high-level plays have been demonstrated by players, but among them the third goal by the Left Wing (LW) of Fujieda at  $\eta=0.7$  is the best: At position A the Left-Wing kicks forward along the solid line, which is the left side of the Full Back (FB) of the opponent team, but she dushes to chase the ball leaving the opponent Full Back behind. Then, immediately after she controls the ball again at position B, she dribbles the ball to set the ball for preparing the ideal shoot at position C. The Left Wing kicks the ball in order to get the goal, and this ball takes her imaged course to go into the far side net of the opponent team goal flying over the Goal Keeper. This goal becomes the third goal of Fujieda against Sakuyo. The present authors are strongly impressed by this goal, for this goal demonstrates her superior creativity, exquisite body balance, excellent skill of shoot and instant adequate decision for play. It is believed that she must be a promising soccer player in Japan.



**Figure 2: The sketch of the third goal by the Left Wing of Fujieda at  $\eta=0.7$ .**  
 ●: Left Wing of Fujieda, ●: Full Back of Sakuyo

### DISCUSSION

The Final, Fujieda vs. Sakuyo will be discussed with reference to information dynamic model, which is expressed by

$$\xi = \eta^m, \tag{4}$$

where  $\xi$  is certainty of game outcome,  $\eta$  is normalized time, and  $m$  is a positive real number. For full account of the information dynamic model, refer to Appendix 2. Figure 1 shows the relation between certainty of game outcome  $\xi$  and normalized time  $\eta$ , where game data of Fujieda vs. Sakuyo together with two curves of the information dynamic model  $\xi = \eta^m$ , where  $m = 2$ , and  $4$ , have been plotted concurrently. It may be interesting to point out the fact that starting from the kick off, this game proceeds almost along with first model curve  $\xi = \eta^2$ , but from  $\eta = 0.743$ , it deviates from this curve to second model curve  $\xi = \eta^4$  but without approaching to the later curve,  $\xi$  increases with  $\eta$  until the end. Nakagawa & Minatoya [8] has proposed a new notion coined 'game point', which is the cross point between certainty of game outcome  $\xi$  and uncertainty of game outcome  $\zeta$ . It is considered that once time exceeds to the game point, one team (or player) gets the safety lead against the other team (or player), and thus it is valuable to discuss the game point in interpreting the game. Let us discuss the effect of Hakusan FC's goal and shoot on the game in terms of game point.

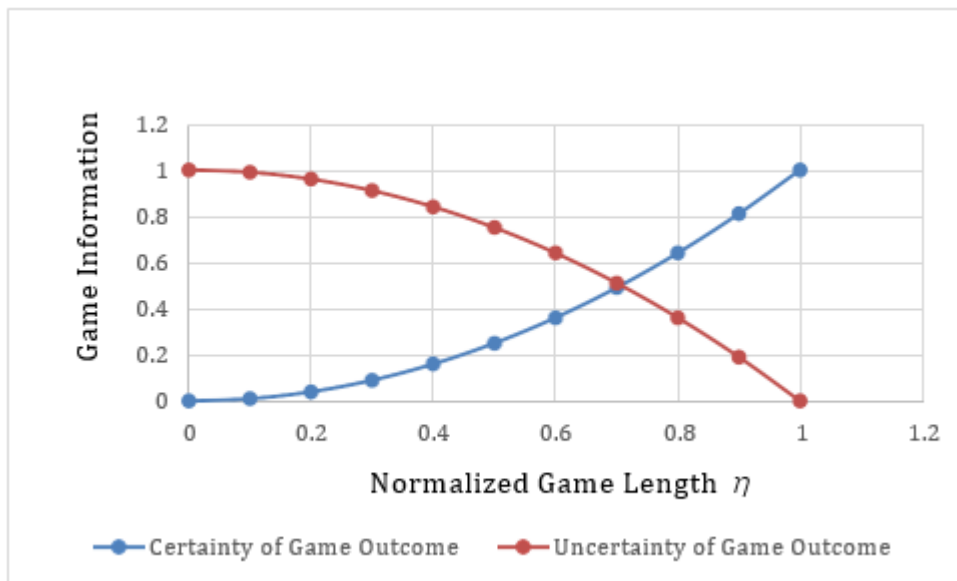


Figure 3: Information of game outcome against normalized time  $\eta$

Figure 3 shows the relation between information of game outcome and normalized time  $\eta$ , where  $\xi = \eta^2$  is the first curve for certainty of game outcome, and  $\zeta = 1 - \eta^2$  is the curve for uncertainty of game outcome. At the game point, certainty of game outcome  $\xi$  is equal to uncertainty of game outcome  $\zeta$ , and both of the curves are taking the common value of 0.5. It can be noted in Figure 3 that in this case the game point is at  $\eta \approx 0.7$ . Thus, if this game can be modelled by the curve  $\xi = \eta^2$  from  $\eta = 0$  to  $1$ , the game outcome of Fujieda's win is predicted at about 0.7(70%) of the total time of  $1$ . It is clear that the game point is significant, for once the game passes it, the team that has the advantage can be guaranteed its win in 100 %, while the opponent has no hope to win the game.

Let us now discuss choice and assessment of evaluation function scores in soccer. There is no question that the choice and assessment are critical to do valuable analyses for supervisor, coach, player or fan. Candidates of evaluation function score in soccer may be goal, shoot, penalty kick, free kick or corner kick, but the assessor(s) is required to decide which one should be chosen and counted as the evaluation function score among the candidates. That is, in case of penalty kick match, evaluation function score of a successful kick must be counted as one, while that of an unsuccessful kick is counted as zero. Furthermore, if course of the ball is out of the goal mouth, and/or speed of the kicked ball is too slow, evaluation function score of a shoot or free kick may be counted as zero. On one hand, even if course of the ball is out of the goal mouth, a strong shoot and/or free kick towards the goal mouth may be counted as one. Whether a free kick is counted as evaluation function score or not, also depends on the case that it is direct or indirect. A direct free kick should be treated as a shoot, but an indirect free kick may not. In case of corner kick, it is known that sometimes the kicked ball goes into the goal directly either by the rotation of ball, wind, or own goal. When the kicked ball reaches at or near the goal area, it is, therefore, necessary to judge carefully whether it should be counted one or zero as evaluation function score. It is realized that though in soccer goal is quite important and unique, consideration of the other candidates such as shoot, penalty kick, free kick or corner kick is essential for simulating and modelling the game faithfully.

It may be interesting to point out that the information dynamic model possesses a potential to predict game outcome, where information of game outcome is expressed as depending on time. Using initial conditions such as team (player)' ranking, and/or record, value of the parameter 'm' of the information dynamic model in (4) may be obtained before a game starts. It is clear that once this value is provided, the game will proceed along one of the model curves as plotted in Figure 5 from the start to the end. The information dynamic model is applicable to predict future trends in social problems such as GDP (Gross Domestic Product), population, temperature or effect of medicine, or educational attainment, where current information such as an inclination angle of the relevant curves is critical.

## CONCLUSIONS

In this section, new knowledge and insights obtained through the present study have been summarized as follows, A soccer game have been successfully simulated and modelled in terms of advantage and certainty of game outcome depending on time. In the data analyses, as evaluation function scores, in addition to goal, it is realized that consideration of shoot, corner kick, or penalty kick are essential, and thus faithful analyses to the game have been conducted, where proper choice and assessment of evaluation function scores are carefully done during the game. The refined game information depending on the time makes possible for supervisor, coach or player to reflect the past game and to prepare for the future game by reviewing the time history of the advantage, certainty of game outcome or game point in the past games, so that the present analyses add a pedagogical value to activities in soccer other than entertainment. This game has been defined as non-categorized game, though it might look as one-sided game. Particularly, it is featured that the third goal of Fujieda is remarkable, so that one of the soccer authorities considers the goal getter, Left Wing of Fujieda, is a promising woman player in WE league, which is the professional Women soccer league, to start this autumn in Japan.

It is suggested that the information dynamic model possesses a potential to predict game outcome before its start.

## REFERENCES

- [1]. Chen, S., Michael, D. *Serious Games: Games that Educate, Train and Inform*. USA, Thomson Course Technology, 2005
- [2]. Kelle, S., Börner, D., Kalz, M., Specht M. Ambient displays and game design patterns. In *WC-TEL710 Proc. of the 5th European Conference on Technology Enhanced Learning Conference on Sustaining TEL from innovation to learning and practice*, 512-517, Springer-Verlag, Berlin, Heidelberg 2010.
- [3]. Lindley, C.A., Sennersten C.C. Game play schemes: from player analysis to adaptive game mechanics. *International Journal of Computer Games Technology*, 7 pages, Article ID216784, 2008
- [4]. Lindley, C.A., Sennersten C.C. A cognitive framework for the analysis of game play: tasks, schemas and attention theory. In *Proc. of the 28th Annual Conference of the Cognitive Science Society*, 13 pages, 26-29 July, Vancouver, Canada 2006.
- [5]. Fullerton, T., Swain, C., Hoffman. S. *Game Design Workshop: Designing, Prototyping, and Play-testing Games*. CMP Books, San Francisco, New York & Lawrence, 2004.
- [6]. Zagal, J., Mateas, M., Fernandez-Vara C., Hochhalter, B., Lichi N. Towards an ontological language for game analysis. In: S. de Castell & J.Jenson(eds.), *Changing views: Worlds in play: Selected papers of DIGRA 2005*(pp.3-14), Vancouver, British Columbia, Canada: Digital Games Research Association. 2005
- [7]. Lundgren, S., Björk S. Describing computer-augmented games in terms of interaction. Paper presented at *Technologies for Interactive Digital Storytelling and Entertainment*, Darmstadt, Germany, 2003.
- [8]. Nakagawa, T.R.M., Minatoya, H. Information dynamics in judo. *Research Journal of Budo*, 47(1), 29-45, 2014.
- [9]. Iida, H., Nakagawa, T., Spoerter, K. Game information dynamic models based on fluid mechanics. *Entertainment and Computing*, 3, 89-99, 2012.
- [10]. Iida, H., Nakagawa, T., Nossal, N. Certainty of patient survival with reference to game information dynamic model. *Open Journal of Preventive Medicine*, 2(4), 490-498, 2012.
- [11]. Nakagawa, T., Nakagawa, A. A safety lead curve and entertainment in games. *International Review of Social Sciences and Humanities*, 7(1), 91-103, 2014.
- [12]. Nakagawa, T.R.M., Iida, H. Three game patterns. *International Journal of Research Studies in Computer Sciences and Engineering*, 1(1), 1-12, 2014.
- [13]. Nakagawa, T., Iida, H., Morichika, H., Ohura, S., Yonenaga, K. Human and interaction in Shogi. *Research Journal of Computation and Mathematics*, 2(1), 7-13, 2014
- [14]. Nakagawa, T., Nakagawa, A. Information dynamics analyses on American Foot Ball. *Social Sci J*. 6(1), 26-40, 2020.
- [15]. Neumann, J., von Morgenstern, O. *Theory of Games and Economics Behavior*. Princeton University Press, 1944.
- [16]. Blasius, H. *Grenzschichten Flüssigkeiten mit kleiner Reibung*. Inaugural- Dissertation zur Erlangung der Doktorwürde der Hohen Philosophischen Fakultät Georgia Augusta zur Göttingen, Druck von B.G. Teubner in Leipzig, 1907.
- [17]. Prandtl L. *Über Flüssigkeitsbewegung bei sehr kleiner Reibung*. Verhandlungen des 3rd Internationalen Mathematiker-Kongresses, Heidelberg, Leipzig: Teubner, 1904
- [18]. Schlichting H. *Boundary-Layer Theory* (translated by J. Kestin), McGraw-Hill, New York, 6th Edition, 1968



[19]. Solso, R. Cognition and the Visual Arts. MIT Press, Cambridge, 1994

### Appendix 1 Game Pattern

Let us define game patterns, for we often encounter the typical patterns such as see-saw, balanced, and one-sided games in baseball, soccer, American football, boxing, chess, horse race, judo, or sumo. However, there has been no definition for game patterns, so that it may be worth proposing it for the promotion of understanding games. First of all, games are categorized as see-saw, balanced, one-sided and non-categorized, and it is proposed that these are defined, respectively, as follows,

- a) See-saw game: Sign in advantage  $\alpha$  alters over 3 times during the game and the peak value of  $\alpha$  at each the 3 periods must greater (smaller) than 0.05(-0.05).
- b) Balanced game: Absolute value of advantage  $|\alpha|$  is always smaller than 0.05 during the game:  $|\alpha| < 0.05$  for  $0 \leq \eta \leq 1$
- c) One-sided game: Advantage  $\alpha$  is equal or greater (smaller) than 0.05(-0.05) over the 90% of the total time.
- d) Non-categorized game: All of the rest games other than the above three patterns.

### Appendix 2 Information Dynamic Model

Currently, information dynamic model only makes it possible to treat and identify game progress patterns depending on the time. In this model, information of game outcome is expressed as a simple analytical function depending on the time, where information of game outcome is certainty of game outcome. In this Appendix, the information dynamic model has been introduced.

#### Modelling Procedure

The modeling procedures of information dynamics based on fluid mechanics are summarized as follows:

- a) Assume a flow problem as the information dynamic model and solve it.
- b) Get the solutions, depending on the position (or time).
- c) Examine whether any solution of the problem can correspond to game information.
- d) If so, visualize the assumed flow with some means. If not, return the first step.
- e) Determine the correspondence between the flow solution and game information.
- f) Obtain the mathematical expression of the information dynamic model.

The information dynamic model will be constructed by following the above procedures step by step.

- a) Let us assume flow past a flat plate at incident angle of zero as the information dynamic model (Figure 4).

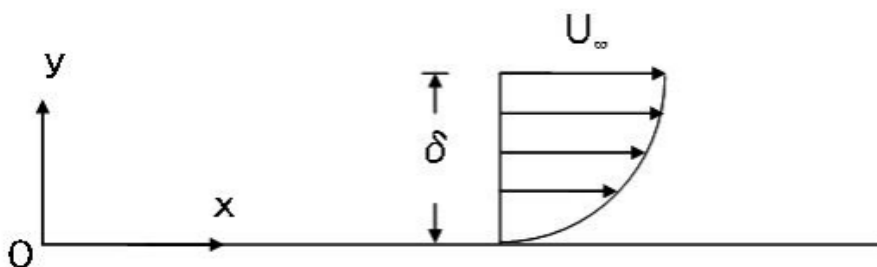


Figure 4 Definition sketch of flow past a flat plate at incident angle of zero.

An example of the application of the boundary-layer equations, which is the simplified version of Navier-Stokes equations [18], is afforded by the flow along a very thin flat plate at incident angle of zero. Historically this is the first example illustrating the application of Prandtl's boundary layer theory [17]; it has been discussed by Blasius [16] in his doctoral thesis at Göttingen. Let the leading edge of the plate be at  $x=0$ , the plate being parallel to the  $x$ -axis and infinitely long downstream, as shown in Figure 4. We shall consider steady flow with a free-stream velocity  $U$ , which is parallel to the  $x$ -axis. The boundary-layer equations [17,18] are expressed by

$$u \cdot \partial u / \partial x + v \cdot \partial u / \partial y = -1/\rho \cdot dp/dx + \nu \partial^2 u / \partial y^2, \quad (5)$$

$$\partial u / \partial x + \partial v / \partial y = 0 \quad (6)$$

$$y = 0: u = v = 0; \quad y = 1: u = U, v = 0. \quad (7)$$

where  $u$  and  $v$  are velocity components in the  $x$ - and  $y$ - directions, respectively,  $\rho$  the density,  $p$  the pressure and  $\nu$  the kinematic viscosity of the fluid. In the free stream by using the conditions at the outer edge of the boundary layer in (7), we obtain

$$U \cdot dU/dx = -1/\rho \cdot dp/dx. \quad (8)$$

The free-stream velocity  $U$  is constant in this case, so that  $dp/dx=0$ , and so  $dp/dy=0$ . Since the system under consideration has no preferred length, it is reasonable to suppose that the velocity profiles at varying distances from the leading edge are similar to each other, which means that the velocity curves  $u(y)$  for varying distances  $x$  can be made identical by selecting suitable scale factors for  $u$  and  $y$ . The scale factors for  $u$  and  $y$  appear quite naturally as the free-stream velocity,  $U$  and the boundary-layer thickness,  $\delta(x)$ . Hence, the velocity profiles in the boundary-layer can be written as

$$u/U = f(y/\delta). \quad (9)$$

Blasius [15] has obtained the solution in the form of a series expansion around  $y/\delta = 0$  and an asymptotic expansion for  $y/\delta$  being very large, and then the two forms being matched at a suitable value of  $y/\delta$ .

- a) The similarity of velocity profile is here accounted for by assuming that function  $f$  depends on  $y/\delta$  only, and contains no additional free parameter. The function  $f$  must vanish at the wall ( $y = 0$ ) and tend to the value of 1 at the outer edge of the boundary-layer ( $y = \delta$ ), in view of the boundary conditions for  $f(y/\delta) = u/U$ .

When using the approximate method, it is expedient to place the point at which this transition occurs at a finite distance from the wall, or in other words, to assume a finite boundary-layer thickness  $\delta(x)$  in spite of the fact that all exact solutions of boundary-layer equations tend asymptotically to the free-stream associated with the particular problem. The "approximate method" here means that all the procedures are to find approximate solutions to the exact solution. When writing down an approximate solution of the present flow, it is necessary to satisfy certain boundary condition for  $u(y)$ . At least the no-slip condition  $u = 0$  at  $y = 0$  and the condition of the continuity when passing from the boundary-layer profile to the free-stream velocity, so that  $u = U$  at  $y = \delta$ , must be satisfied.

The following velocity profile satisfies all of the boundary conditions as the tentative solutions (Ansatz Lösungen in German) on the flow past a flat plate at incident angle of zero,

$$u/U = (y/\delta)^m, \quad (10)$$

where  $m$  is positive real number. Eq. (10) is solutions for the assumed flow. This is heuristically discovered, and represents a group of the approximate solutions with each different value of  $m$ . When  $m=1$ , (10) becomes the exact solution, but all of the rest solutions are approximate solutions to exact solutions, respectively.

- a) Let us examine whether this solution is game information or not. Such an examination immediately provides us that the non-dimensional velocity varies from 0 to 1 with increasing the non-dimensional vertical distance  $y/\delta$  in many ways as the non-dimensional information, so that these solutions can be game information. However, validity of this conjecture shall be confirmed by the relevant data.
- b) Imagine that the assumed flow is visualized with neutral buoyant particles. Motion of the visualized particles is detected by the eye almost instantaneously through the lights and is mapped on our retina first [19], so that during these processes, motion of "fluid particles" is transformed into that of the "information particles" by the light carrying the images of fluid particles. This is why motion of the fluid particles is intact in the physical space, but only the reflected lights, or electromagnetic waves consisting of photons can reach at the retina. The photons are then converted to electrochemical particles and are passed along the visual cortex for further processing in parts of the cerebral cortex [19]. The photons and/or electrochemical particles are considered to be information particles. It is, therefore, natural to expect that the flow in the physical space is faithfully transformed to that in the information space, or brain including eye. During this transformation, the flow solution in the physical space changes into the information solution in the information space.
- c) Proposed are correspondences between the flow and game information, which are listed in Table 2.

**Table 2: Correspondences between flow and game information**

Flow	Game
$u$ : flow velocity	$I$ : current information
$U$ : free stream velocity	$I_0$ : total information
$y$ : vertical co-ordinate	$t$ : current time
$\delta$ : boundary layer thickness	$t_0$ : total time

- a) Considering the correspondences in Table 2, (10) can be rewritten as  $I/I_0 = (t/t_0)^m$  (11)

Introducing the following normalized variables in (11),

$$\xi = I/I_0 \text{ and } \eta = t/t_0,$$

we finally obtain the analytical expression of the information dynamic model as

$$\xi = \eta^m, \quad (12)$$

where  $\xi$  is the normalized information,  $\eta$  the normalized time, and  $m$  is a positive real number.

Figure 4 shows the relation between certainty of game outcome  $\xi$  and normalized time  $\eta$ , where a total of 9 model curves have been plotted concurrently. This figure clearly suggests versatility of this model (12), for each of the curve represents a game. Thus, this model can represent any game in principle, where each of games takes a unique value of  $m$ . The smaller the strength difference between both teams (or players) is, the greater the value of  $m$ , and vice versa.

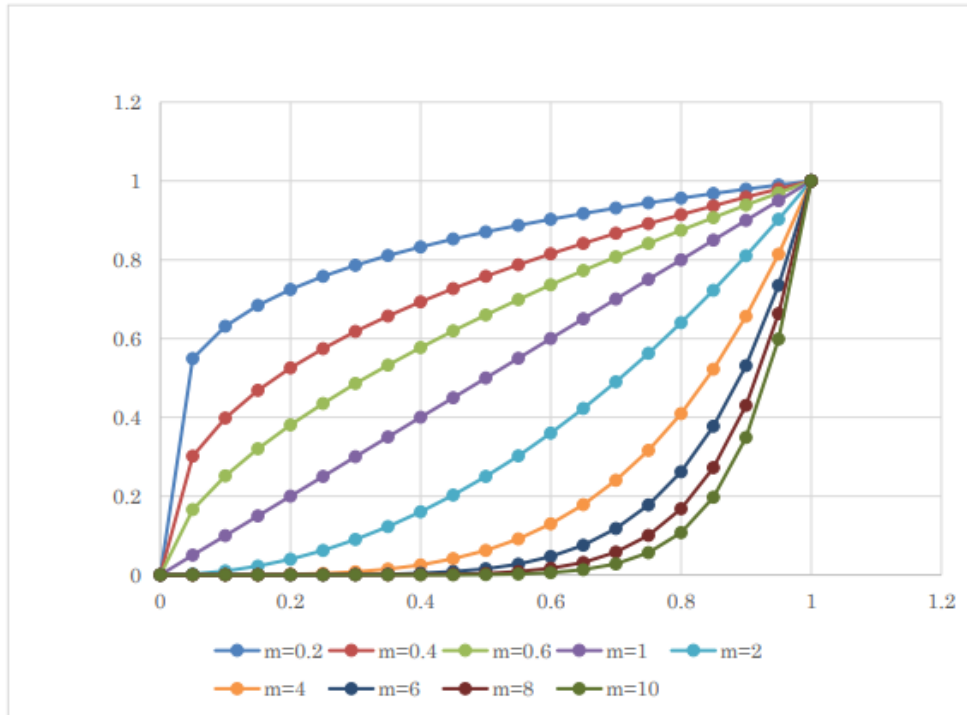


Figure 5 Certainty of game outcome  $\xi$  against normalized time  $\eta$ .