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On the Neutrosophic Algebraic Structures Involving Similarity and the Symmetry Properties on the Neutrosophic Interval Probability

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Abstract:
The neutrosophic interval statistical number (NISN) has been known to be very useful in expressing the interval values under indeterminate environments. One of the essential and so important useful as tools for measuring the degree of similarity between sets of given objects is the similarity measure. In this paper, neutrosophic numbers as well as the generalized Dice similarity measure for neutrosophic numbers for two sets are defined after which the axioms of fuzziness similarity and symmetry satisfying the NISN the properties were proved.

Keywords: Similarity, fuzziness, neutrosophic interval, symmetry Properties, generalized dice similarity measure (GDSM)

INTRODUCTION

The hesitant fuzzy linguistic information has been very useful as much as also being very applicable in finding lasting solutions and recommendations to multiple problems. Such like the attributes in decision making processes are often being finally resolved. As such, some algorithms can be developed in order for the utilization of the generalized Dice similarity measures giving required solutions. The renowned concept of neutrosophic statistics was founded and developed by the efforts of Prof. Dr. Florentin Smarandache, from the University of New Mexico, United States. This was originally started in1998, and then followed much later for further developments on it in 2014. Then, there was some introduction of the Neutrosophic

Descriptive Statistics (NDS). Sequel to this was the efforts put up by Prof. Dr. Muhammad Aslam, from the King Abdulaziz University, Saudi Arabia, who introduced the neutrosophic inferential statistics (NIS), Neutrosophic Applied Statistics (NAS), and Neutrosophic Statistical Quality Control (NSQC). in 2018. As it were, the Neutrosophic Statistics seems to be a generalization of Interval Statistics. This among others, is because of, while Interval Statistics is based on Interval Analysis, Neutrosophic Statistics is much based on Set Analysis (meaning every other kind of sets, and not only intervals as usual). The Neutrosophic Statistics is more elastic in style than the well-known and conversant Classical Statistics. Hence, if all the data and inference methods are determinate, then the Neutrosophic Statistics coincides with the Classical Statistics.

But, since in our world generally, we have more indeterminate data than those that are determinate. Therefore, there seems to be more neutrosophic statistical procedures which are needed than classical ones. The Dice similarity coefficient is a statistical tool. What it does is to measure the similarity between two given sets of data. It can also be referred to as the Sørensen–Dice index. (See [1]) We can also call it simply as the Dice coefficient. Some functions of which the similarity degree is expressed which involves certain items can be used in physical entities and phenomenon such as anthropology, automatic classification, psychology, ecology, information retrieval, citation analysis, numerical taxonomy and patterns recognition. The degree of dissimilarity or similarity between any given sets of objects plays a very important and vital role. space, most especially, in vector the cosine Jaccard, as well as the Dice similarity measures are often very useful in citation analysis, information retrieval, and also in automatic classification. In many cases, the Dice similarity measures as well as the asymmetric measures (also known as the projection measures) happen to be the special cases in some parameter values.

THE METHOD AS ADOPTED ON THE INTERVAL PROBABILITY AND THE NEUTROSOPHIC STATISTICAL NUMBER

The concepts of neutrosophic statistics seems to extend the classical statistics. It deals with set values instead of crisp values. In most of the classical statistics equations and formulas, one simply replaces several numbers by sets. Consequently, instead of operations with numbers, one uses operations with sets. One normally replaces the parameters that are indeterminate (imprecise, unsure, and even completely unknown).

Looking at the Neutrosophic interval statistical number, it is supposed to be a form of interval sets or range of certain values or other common entities. In most cases, it can be in the form of intervals (such kinds of intervals can be of the form known as closed, half closed, half open or open) and mostly represented as: [x, y], [x, y), (x, y], and (x, y), where x and y are the usual real numbers.

The neutrosophic interval probability (NIP has been defined in a range given by: $[V^L, V^U]$ of individuals in the given sample. (See [4, and 5]) The form of a NIP form can be clearly expressed as follows: D = < $[V^L, V^U]$, (D_T, D_I, D_F)>, where, are the true probability is given by D_T, while that of indeterminate, and false probabilities are given as D_I and D_F. Each of these could be found respectively in the range of the determinate, indeterminate, and failure. Now for each trial data, we have that the neutrosophic interval probability defined in an equational form as follows:

$$D_T = \frac{n_T}{n},\ D_T = \frac{n_T}{n} \text{ and } D_T = \frac{n_T}{n}$$

Here, n is the total number of the individuals totals n. are the Some number of samples falls in the interval $[v_m - \sigma, v_m + \sigma]$. This is denoted by n_T.For n_I, the interval is given by: $[v_m - 3\sigma, v_m - \sigma]$ and for n_F, it is given by : $(v_m + \sigma, v_m + 3\sigma)$, which is the number of the rest samples. Also, v_m is the statistical mean value while the standard deviation is represented by σ. The addition of all the probabilities equals 1. Efforts were intensified to clarify the proof the axioms of fuzziness similarity and symmetry satisfying the NISN the properties were proved (see also, [6, 7, and 8]).

On the Numerical Neutrosophic Numbers

Here, (in this case, in a way to approximate the imprecise data the indeterminacy "I" is always given as real subsets. Hence, making it more general than the interval. This is because "I" may be given as any subset. Take instance, N = 8 + 6I, where "I" is in the discrete hesitant subset {0.4, 0.9, 6.4, 55.6} in this case having only four elements, which is not part of interval analysis (such as in statistics). But for the interval statistics, the interval [0.4, 55.6] is taken so as to include those given numbers which fall within the given range of the intervals. But then, with this, the level of the uncertainty seems to increase so much considerably. Now, in cases where the "I" is an interval given as I = [I_1, I_2], with $I_1 \leq I_2$, we are going to have that N = x + yI. This actually coincides with the interval which is given by: N = [x + yI_1, x + yI_2]. (Please, see [10] for more details)

THE FUZZINESS, SIMILARITY AND THE SYMMETRY PROPERTIES

The Fuzziness Condition

$$A1.\ 0 \leq E(R_A, R_B) \leq 1$$

Definition 1: (see [3]):

For a classical Neutrosophic Number, the standard form can be expressed as $a+bI$. Also, a as well as b are real number coefficients. I is the indeterminacy, whence $0 \cdot I = 0$ and $I^2 = I$ are both true. Hence, we have that $I^n = I$, and this is true for each of the positive integer given by *n*. Now, we call $a+bI$ the Neutrosophic Real Number whenever the two arbitrary coefficients *a* as well as *b* are real numbers.

Furthermore, it should be well noted here, that we have literal neutrosophic numbers, a+bI, where I = letter, and I^2 = I, and numerical neutrosophic numbers, a+bI, where I = a real subset (normally interval). But herein $I^2 \neq I$.

Take for example, the following, as quoted from [3]:

a. $[10.2, -8]^2 = [-10.2, -8] \cdot [-10.2, -8] = [64, 104.04]$.
b. $[-14.25, -9]^2 = [(-9)^2, (-14.25)^2] = [81, 203.0625]$
c. $[13.8, 16]^2 = [13.8^2, 16^2] = [190.44, 256]$
d. $[1.8, 10]^2 = [1.8^2, 10^2] = [3.24, 100]$

Clearly, from this analysis, $I^2 \neq I$.

Definition 2: (see [9]):

An important measure about the similarity in between two objects can sometimes be referred to as the Similarity measures (SMs). In this case, a special kind of such measures is often applied to be used mostly in comparing objects is the generalized dice similarity measure (GDSM).

Definition 3: (see [3]):

Suppose that $R_A = a_A + b_A I$......... (i) and $R_B = a_B + b_B I$.........(ii) are neutrosophic numbers, such that each of a_A, b_A, a_B, and $b_B \geq 0$. We define a generalized dice similarity measure in this manner in between R_A and R_B:

$$E(R_A, R_B) = \frac{2R_A.R_B}{|R_A|^2 + |R_B|^2} = L \text{ (say)}$$

Then,

$$L = 2\text{x}\frac{(a_A + \inf(b_A I))(a_B + \inf(b_B I) + (a_A + sup(b_A I))(a_B + \sup(b_B I)}{(a_A + \inf(b_A I)^2 + (a_A + \sup(b_A I)^2 + (a_B + \inf(b_B I)^2 + (a_B + \sup(b_B I)^2}$$

(Note that each of$(a_A + \inf(b_A I))$, $(a_B + \inf(b_B I)$, $(a_A + sup(b_A I))$, and $(a_B + \sup(b_B I)$ is a real number since each of the components such as a_A, $\inf(b_A I)$, a_B, $\inf(b_B I)$,etc. is rea number).

We have,

$$L = \frac{2(a_A + \inf(b_A I))(a_B + \inf(b_B I) + 2(a_A + sup(b_A I))(a_B + \sup(b_B I)}{[(a_A + \inf(b_A I)^2 + (a_B + \inf(b_B I)^2\ ] + [(a_A + \sup(b_A I)^2 + (a_B + \sup(b_B I)^2]}$$

$$= \frac{2(a_A + \inf(b_A I))(a_B + \inf(b_B I) + 2(a_A + sup(b_A I))(a_B + \sup(b_B I)}{2(a_A + \inf(b_A I))(a_B + \inf(b_B I) + 2(a_A + sup(b_A I))(a_B + \sup(b_B I) + [(a_A + \inf(b_A I)) + (a_B + \inf(b_B I)]^2 + [(a_A + sup(b_A I)) + (a_B + \sup(b_B I)]^2}$$

Dividing through by $2(a_A + \inf(b_A I))(a_B + \inf(b_B I) + 2(a_A + sup(b_A I))(a_B + \sup(b_B I)$

We have: $\frac{1}{1 + [(a_A + \inf(b_A I)) + (a_B + \inf(b_B I)]^2 + [(a_A + sup(b_A I)) + (a_B + \sup(b_B I)]^2}$(k)

(Here, it should be observed that since $(X + Y)^2 = X^2 + Y^2 + 2XY$, it implies that $X^2 + Y^2 = (X + Y)^2 - 2XY$)

Obviously, this is a positive number which is greater than 0. Hence, this satisfies the left-hand side of the inequality. i.e., $0 \leq L = E(R_A, R_B)$

Now, to show that L is less or equal to 1, observe that the denominator is positive since the addition of positive numbers is positive, whence the square of any real number is positive. We thus prove this by contradiction. Assume that $L \nleq 1$.

Let $X^2 = [(a_A + \inf(b_A I)) + (a_B + \inf(b_B I)]^2 + [(a_A + sup(b_A I)) + (a_B + \sup(b_B I)]^2$We have that $L = \frac{1}{1+X^2} > 1$, we have that $1 > 1 + X^2 \Longrightarrow X^2 < 0$. A contradiction ($\Longrightarrow\Longleftarrow$). Hence, $0 \leq E(R_A, R_B) \leq 1$. This satisfies A1■

The Similarity Condition
A2. $E(R_A, R_B) = 1$ if $R_A = R_B$

Proof:
($\Leftarrow$) Assume that $R_A = R_B = R = a + bI$

Then, by definition,

$$E(R_A, R_B) = E(R, R) = \frac{2R_A.R_B}{|R_A|^2 + |R_B|^2} = \frac{2R.R}{|R|^2 + |R|^2}$$

$$2\text{x}\frac{(a + \inf(bI))(a + \inf(bI) + (a + sup(bI))(a + \sup(bI)}{(a + \inf(bI)^2 + (a + \sup(bI)^2 + (a + \inf(bI)^2 + (a + \sup(bI)^2}$$

$$= \frac{2(a + \inf(bI))(a + \inf(bI) + 2(a + sup(bI))(a + \sup(bI)}{(a + \inf(bI)^2 + (a + \sup(bI)^2 + (a + \inf(bI)^2 + (a + \sup(bI)^2}$$

$$= \frac{2(a + \inf(bI))(a + \inf(bI) + 2(a + sup(bI))(a + \sup(bI)}{2(a + \inf(bI)^2 + 2(a + \sup(bI)^2}$$

$$= \frac{2(a + \inf(bI)^2 + 2(a + \sup(bI)^2}{2(a + \inf(bI)^2 + 2(a + \sup(bI)^2} = 1.$$

($\Rightarrow$) Assume that $E(R_A, R_B) = \frac{2R_A.R_B}{|R_A|^2 + |R_B|^2} = 1$. Then, $2R_A.R_B = |R_A|^2 + |R_B|^2$

$$\Rightarrow 2(a_A + \inf(b_A I))(a_B + \inf(b_B I) + (a_A + sup(b_A I))(a_B + \sup(b_B I)$$

$$=(a_A + \inf(b_A I)^2 + (a_A + \sup(b_A I)^2 + (a_B + \inf(b_B I)^2 + (a_B + \sup(b_B I)^2$$

$$\Rightarrow 2(a_A + \inf(b_A I))(a_B + \inf(b_B I) + 2(a_A + sup(b_A I))(a_B + \sup(b_B I)$$

$$=(a_A + \inf(b_A I)^2 + (a_B + \inf(b_B I)^2 + (a_A + \sup(b_A I)^2 + (a_B + \sup(b_B I)^2$$

Equating components, we have,

$$2(a_A + \inf(b_A I))(a_B + \inf(b_B I) = (a_A + \inf(b_A I)^2 + (a_B + \inf(b_B I)^2 \text{ And} 2(a_A + sup(b_A I))(a_B + \sup(b_B I) = (a_A + \sup(b_A I)^2 + (a_B + \sup(b_B I)^2$$

$$\Rightarrow (a_A + \inf(b_A I)) = (a_B + \inf(b_B I) = (a + \inf(bI) \text{ and } (a_A + sup(b_A I)) = (a_B + \sup(b_B I) = (a + \sup(bI) \text{ (say)}$$

$\Rightarrow R_A = R_B$ with the condition that:

$$\begin{cases} 2(a_A + \inf(b_A I))(a_B + \inf(b_B I) = (a_A + \inf(b_A I)^2 + (a_B + \inf(b_B I)^2 \\ \text{And} 2(a_A + sup(b_A I))(a_B + \sup(b_B I) = (a_A + \sup(b_A I)^2 + (a_B + \sup(b_B I)^2 \end{cases}$$

This satisfies A2 ■

The Symmetry Condition
P3. E (R_A, R_B) = E (R_B, R_A)

Proof:
We have that,

$$E(R_A, R_B) = \frac{2R_A . R_B}{|R_A|^2 + |R_B|^2}$$

$$= 2\text{x}\frac{(a_A + \inf(b_A I))(a_B + \inf(b_B I) + (a_A + sup(b_A I))(a_B + \sup(b_B I)}{(a_A + \inf(b_A I)^2 + (a_A + \sup(b_A I)^2 + (a_B + \inf(b_B I)^2 + (a_B + \sup(b_B I)^2}$$

$$= 2\text{x}\frac{(a_B + \inf(b_B I)(a_A + \inf(b_A I)) + (a_B + \sup(b_B I)(a_A + sup(b_A I))}{(a_B + \inf(b_B I)^2 + (a_B + \sup(b_B I)^2 + (a_A + \inf(b_A I)^2 + (a_A + \sup(b_A I)^2}$$

$$= \frac{2R_B . R_A}{|R_B|^2 + |R_A|^2} = E(R_B, R_A). \text{ This satisfies A3 ■}$$

The Fuzzy Condition
A4. $0 \leq E(A, B) \leq 1$

Definition 3: (see [2]):
Let A = {R_{A1}, R_{A2}, ..., R_{An}} and B = {R_{B1}, R_{B2}, ..., R_{Bn}}be two sets which are neutrosophic numbers, and that $R_{Ak} = a_{Ak} + b_{Ak}I$, $R_{Bk} = a_{Bk} + b_{Bk}I$ such that (k = 1, 2, ..., n). In addition, each of a_{Aj}, b_{Aj}, a_{Bj} and $b_{Bj}$is positive. i.e., ≥ 0. Then, the number which is called the generalized Dice similarity measure in between the sets A and B can are usually being found by using the expansion given as:

$$E(A, B) = \sum_{j=1}^{n} w_j \frac{2R_{Aj} . R_{Bj}}{|R_{Aj}|^2 + |R_{Bj}|^2}$$

$$= 2\sum_{j=1}^{n} w_j \frac{(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I))+(a_{Aj}+sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I))}{(a_{Aj}+\inf(b_{Aj}I)^2+(a_{Aj}+\sup(b_{Aj}I)^2+(a_{Bj}+\inf(b_{Bj}I)^2+(a_{Bj}+\sup(b_{Bj}I)^2}$$

$$= 2\left(w_1 \frac{(a_{A1}+\inf(b_{A1}I))(a_{B1}+\inf(b_{B1}I)+(a_{A1}+sup(b_{A1}I))(a_{B1}+\sup(b_{B1}I)}{(a_{A1}+\inf(b_{A1}I)^2+(a_{A1}+\sup(b_{A1}I)^2+(a_{B1}+\inf(b_{B1}I)^2+(a_{B1}+\sup(b_{B1}I)^2}\right)$$

$$+ 2\left(w_2 \frac{(a_{A2}+\inf(b_{A2}I))(a_{B2}+\inf(b_{B2}I)+(a_{A2}+sup(b_{A2}I))(a_{B2}+\sup(b_{B2}I)}{(a_{A2}+\inf(b_{A2}I)^2+(a_{A2}+\sup(b_{A2}I)^2+(a_{B2}+\inf(b_{B2}I)^2+(a_{B2}+\sup(b_{B2}I)^2}\right)$$

$$+\cdots + 2\left(w_k \frac{(a_{Ak}+\inf(b_{Ak}I))(a_{Bk}+\inf(b_{Bk}I)+(a_{Ak}+sup(b_{Ak}I))(a_{Bk}+\sup(b_{Bk}I)}{(a_{Ak}+\inf(b_{Ak}I)^2+(a_{Ak}+\sup(b_{Ak}I)^2+(a_{Bk}+\inf(b_{Bk}I)^2+(a_{Bk}+\sup(b_{Bk}I)^2}\right)$$

$$+\cdots 2\left(w_n \frac{(a_{An}+\inf(b_{An}I))(a_{Bn}+\inf(b_{Bn}I)+(a_{An}+sup(b_{An}I))(a_{Bn}+\sup(b_{Bn}I)}{(a_{An}+\inf(b_{An}I)^2+(a_{An}+\sup(b_{An}I)^2+(a_{Bn}+\inf(b_{Bn}I)^2+(a_{Bn}+\sup(b_{Bn}I)^2}\right)$$

Now, let

$$Q = 2\sum_{j=1}^{n} w_j \frac{(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I))+(a_{Aj}+sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I))}{(a_{Aj}+\inf(b_{Aj}I)^2+(a_{Aj}+\sup(b_{Aj}I)^2+(a_{Bj}+\inf(b_{Bj}I)^2+(a_{Bj}+\sup(b_{Bj}I)^2}$$

$$= \sum_{j=1}^{n} w_j \frac{2(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I))+2(a_{Aj}+sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I))}{2(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I))+2(a_{Aj}+sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I))+ [(a_{Aj}+\inf(b_{Aj}I))+(a_{Bj}+\inf(b_{Bj}I))]^2+[(a_{Aj}+sup(b_{Aj}I))+(a_{Bj}+\sup(b_{Bj}I))]^2}$$

Dividing through by

$$2(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I))+2(a_{Aj}+sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I))$$

We have that:

$$Q = \sum_{j=1}^{n} w_j \frac{1}{1+[(a_{Aj}+\inf(b_{Aj}I))+(a_{Bj}+\inf(b_{Bj}I))]^2+[(a_{Aj}+sup(b_{Aj}I))+(a_{Bj}+\sup(b_{Bj}I))]^2}$$

(Here, $\sum_{j=1}^{n} w_j = w_1 + w_2 + w_3 + w_n = 1$) (*)

And clearly, the fraction is a positive number which is greater than 0. Hence, this satisfies the left-hand side of the inequality. i.e., $0 \leq Q = E(A, B)$

Now, to show: $Q \leq 1$, We thus prove this by contradiction. Assume that $Q \nleq 1$.

Let

$$Y_j^2 = [(a_{Aj}+\inf(b_{Aj}I))+(a_{Bj}+\inf(b_{Bj}I))]^2+[(a_{Aj}+sup(b_{Aj}I))+(a_{Bj}+\sup(b_{Bj}I))]^2$$

We have that

$$Q = \frac{1}{1+Y_j^2} > 1,$$

we have that

$$1 > 1 + X^2 \Longrightarrow X^2 < 0. \text{ A}$$

We have that

$$Q = \sum_{j=1}^{n} w_j \frac{1}{1+Y_j^2} > 1,$$

we have that

$$1 > 1 + X^2 \Longrightarrow X^2 < 0.\ A$$

$$w_1 \frac{1}{1+Y_1^2} + w_2 \frac{1}{1+Y_2^2} + w_3 \frac{1}{1+Y_3^2} + \cdots + w_n \frac{1}{1+Y_n^2} > 1$$

(And since $w_1 + w_2 + w_3 + w_n = 1$, let $w_j = \frac{1}{x_j^2}$)

We have that

$$Q = \frac{1}{x_1^2}\frac{1}{(1+Y_1^2)} + \frac{1}{x_2^2}\frac{1}{(1+Y_2^2)} + \frac{1}{x_3^2}\frac{1}{(1+Y_3^2)} + \cdots + \frac{1}{x_n^{2(}}\frac{1}{1+Y_n^2)} > 1$$

$$\frac{1}{x_1^2(1+Y_1^2)} + \frac{1}{x_2^2(1+Y_2^2)} + \frac{1}{x_3^2(1+Y_3^2)} + \cdots + \frac{1}{x_n^2(1+Y_n^2)} > 1$$

Definitely, the LHS is not greater than 0. Hence, the initial assumption is false, and thus

$$0 \le Q = \sum_{j=1}^{n} w_j \frac{1}{1+[(a_{Aj}+\inf(b_{Aj}I)) + (a_{Bj}+\inf(b_{Bj}I)]^2 + [(a_{Aj}+sup(b_{Aj}I)) + (a_{Bj}+\sup(b_{Bj}I)]^2} = E(A, B) \le 1$$

This satisfies A4 ■

The Similarity Condition
A5. E (A, B) = 1 provided that A and B are equal

Proof:
(⇐) If we assume that A and B are equal and are equal to R

Then,

$$E(A, B) = E(R, R) = \frac{2R.R}{|R|^2 + |R|^2}$$

$$= 2\sum_{j=1}^{n} w_j \frac{(a_{Rj}+\inf(b_{Rj}I))(a_{Rj}+\inf(b_{Rj}I) + (a_{Rj}+sup(b_{Rj}I))(a_{Rj}+\sup(b_{Rj}I)}{(a_{Rj}+\inf(b_{Rj}I)^2 + (a_{Rj}+\sup(b_{Rj}I)^2 + (a_{Rj}+\inf(b_{Rj}I)^2 + (a_{Rj}+\sup(b_{Rj}I)^2}$$

$$= \sum_{j=1}^{n} w_j \frac{2(a_{Rj}+\inf(b_{Rj}I))^2 + 2(a_{Rj}+\sup(b_{Rj}I)^2}{2(a_{Rj}+\inf(b_{Rj}I)^2 + 2(a_{Rj}+\sup(b_{Rj}I)^2}$$

$$= \sum_{j=1}^{n} w_j = w_1 + w_2 + w_3 + w_n = 1 \text{ by } (*)$$ ■

The Symmetry Condition
A6.E[A, B] = E [B, A]

$$D(A, B) = \sum_{j=1}^{n} w_j \frac{2R_{Aj}.R_{Bj}}{|R_{Aj}|^2 + |R_{Bj}|^2}$$

$$= 2\sum_{j=1}^{n} w_j \frac{(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I)+(a_{Aj}+sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I)}{(a_{Aj}+\inf(b_{Aj}I)^2+(a_{Aj}+\sup(b_{Aj}I)^2+(a_{Bj}+\inf(b_{Bj}I)^2+(a_{Bj}+\sup(b_{Bj}I)^2}$$

$$= 2\sum_{j=1}^{n} w_j \frac{(a_{Bj}+\inf(b_{Bj}I)(a_{Aj}+\inf(b_{Aj}I))+(a_{Bj}+\sup(b_{Bj}I)(a_{Aj}+sup(b_{Aj}I))}{(a_{Bj}+\inf(b_{Bj}I)^2+(a_{Bj}+\sup(b_{Bj}I)^2+(a_{Aj}+\inf(b_{Aj}I)^2+(a_{Aj}+\sup(b_{Aj}I)^2}$$

$$= \sum_{j=1}^{n} w_j \frac{2R_{Bj}.R_{Aj}}{|R_{Bj}|^2+|R_{Aj}|^2} = \text{D (B, A) This satisfies P6} \blacksquare$$

APPLICATIONS

So far, it can be deduced and inferred that the fuzziness, similarity and the symmetry properties on the neutrosophic interval probability is of utmost importance and thus could be made applicable in such kind of similar cases.

CONCLUSION

Finally, the proofs of the Fuzziness, Similarity and The Symmetry Properties on The Neutrosophic Interval Probability have been fully given

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CONFLICTS OF INTEREST

The author hereby declares that there is no competing of interests whatsoever before, during and after the course of the production and publication of this work

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